



Evidence for the Phillips Curve in Three Countries

Aise O'Neil, American University

During the great recession, inflation failed to fall as much as many expected it too (Gordon 2013), (Ball & Mazumder 2018). This development confused policymakers who were using a Phillips Curve model that presumed an unemployment gap should lead to declining inflation. Understanding why previous Phillips Curve models failed in 2008 and finding a Phillips Curve model that explains recent pre-Covid data, could help guide monetary policy decisions in the future.

To explain inflation during and after the Great Recession, Central Bankers suggested well anchored inflation expectations were keeping inflation stable throughout the crisis (Bernanke 2010) (Yellen 2013). If we assume anchored inflation expectations, unemployment falling lower (or higher) relative to potential should still predict inflation falling lower (or higher) relative to expectations. However, because inflation remains anchored and does not follow actual inflation, we should not expect a deflationary (or inflationary) spiral characterized by shifting expected and actual inflation. Hence, anchored expectations explain low but stable inflation in the context of a prolonged period of high unemployment.

The explanation for the anchored-expectations hypothesis is given by the equations 1-3. Equation 1 is the expectations-augmented phillips curve which relates inflation (π), to unemployment (u) and potential unemployment (u^*). Inflation in one period has a 1-1 relationship with expected inflation in later periods. The difference between inflation and expected future inflation is given by an error term (ϵ) and a function of unemployment (u) and potential unemployment (u^*). That function, f , is negative when u is higher the u^* , 0 when u is equal to u^* and positive when u exceeds u^* . Assuming that expectations are backward-looking and based on previous experience such that $E_T(\pi_{T+1}) = \pi_{T-1}$, gives equation 2, which is a traditional accelerationist model that predicted spiraling deflation after the great recession. However, assuming expected inflation can be anchored at a specific level (“A”) gives equation 3 which would predict stable inflation in the event of high sustained unemployment.

$$1: \quad \pi_T = E_T(\pi_{T+1}) + f(u_t, u_t^*) + \epsilon$$

$$2: \quad \Delta\pi_T = f(u_t, u_t^*) + \epsilon$$

$$3: \quad \pi_T = A + f(u_t, u_t^*) + \epsilon$$

Additionally, arguments were made that the structure of the Phillips Curve measured in terms of unemployment shifted and a new, higher unemployment rate was necessary to keep inflation on par with expectations. To back this viewpoint, some pointed to a shifting Beveridge Curve and significant growth among those unemployed for longer than 6 months (Gordon 2013). The longer one is unemployed, the less likely it is they will find employment over a specified period of time (Budd, Levine & Smith 1988). It has long been speculated, and has recently been concluded in a Brookings Paper (Krueger et. al. 2016) that the long term unemployed are less attached to the labor force than other unemployed workers.

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Taking these considerations into account, two researchers wrote first a working paper (Ball & Mazumder 2015) and then a journal article (Ball & Mazumder 2018). They developed a Phillips Curve model which fit well with data, especially compared with other models over the same time periods. This is thanks to three factors. First, they used a non-standard unemployment measure: the level of people unemployed for less than 26 weeks. Secondly, they used survey-based measures of inflation expectation. Thirdly, they used a more stable measure of inflation: median inflation. The research conducted by Ball and Mazumder will be expanded upon in this paper to other datasets, countries, and later dates.

Following their work, inflation failed to rise as much as the unemployment rate's trajectory led some to predict (Hooper, Mishkin, & Sufi 2020). A team of researchers found significant empirical evidence of the total unemployment-based Phillips Curve existing over long periods of time (Hooper, Mishkin, & Sufi 2020). They additionally attempted to find evidence of a concave Phillips Curve, in which declining unemployment, lower than a certain level, has a strong marginal impact on inflation. (Hooper, Mishkin & Sufi 2020) attempted to explain inflation in terms of 5 functions of measured unemployment and estimated long-term unemployment: 1, inflation as a linear function of the unemployment gap; 2, inflation as a linear function of the unemployment gap over the unemployment rate; 3, inflation as a linear function of the unemployment gap over the potential unemployment rate; 4, inflation as a cubic function of the unemployment gap; 5 inflation as a spline function of the unemployment gap. This article will attempt to explain inflation in terms of each of these five functions. Additionally, a sixth function will be added, which will simply be the unemployment rate. Regressing inflation data against unemployment is arguably necessary to fully provide empirical evidence for the Phillips Curve. According to the Congressional Budget Office ("CBO"), "The historical estimate of the NAIRU [Non-Accelerating Inflation Rate of Unemployment] derives from an econometric estimate of a Phillips curve," (2001). This method of measuring potential unemployment is natural and too be expected. However it raises the likelihood that inflation data determines the potential unemployment rate which may cause a Phillips Curve model to artificially fit well with the data, independent of actual unemployment's behavior. Arguably, Ball and Mazumder (2015), guarded against this in their working paper by using the Hodrick-Prescott Filter to determine potential unemployment. Nonetheless, such a filter presents it's own difficulties as it uses future unemployment data to "predict" inflation.

The intuition relied upon by Ball & Mazumder argues one should expect a strong Phillips Curve relationship between inflation, long-term survey based inflation-expectations among experts and the rate of people who are unemployed for less than 26 weeks. They demonstrated this in the case of one grouping of data sets in 2015. This paper will expand their work by testing their assumption in more datasets and countries with data leading up until 2019. The specific data used and means of analysis are presented in section 1. Overall strength of the Phillips Curve relationship is given in section 2. In Section 3, this paper shows that the Phillips Curve model suggested by Ball & Mazumder would have inadequately predicted inflation trends from 2015 to 2019. In Section 4, the comparative ability of measures of labor market slack such as those used in (Hooper, Mishkin and Sufi 2020) in matching the data will be examined. Section 5 is the conclusion section of this paper and gives research and policy implications.

II. Data Collection and Methodology

Ball & Mazumder's original formulation of the short-term unemployment rate was the share of the labor force that was unemployed for less than 26 weeks. However, that rate can be influenced by the portion of the population which is unemployed for more than 26 weeks. In fact, a large contingent of people who are unemployed for just over 26 weeks would reduce the measured short-term unemployment rate, thereby indicating a low level of labor market slack. To address this issue, this paper changes the denominator to be the labor force minus the number of people unemployed for more than 26 weeks. Additionally, for data from Australia, the benchmark was changed to 6 months as weekly unemployment duration data was not available (6 month is pretty close to 26 weeks).

Long term unemployment was measured two ways. One way was a manner employed by Ball & Mazumder in their working paper: applying a Hodrick–Prescott Filter (HP Filter) with a penalty parameter of 16,000 to quarterly data. Another method used estimates of potential unemployment which economists had previously developed for the traditional unemployment measure (“U3”). The second method was applied in their Journal Article: take an estimate for potential U3 unemployment based on the standard measure and multiply it by the average value of your new unemployment rate and divide it by the average value of the U3 rate over the time span you are investigating. Two sources of NAIRU estimates were used. One was Office of Management and Budget (“OMB”) estimates for the US. The other was Organization of Economic Cooperation and Development (“OECD”) estimates for all three countries.

The specific value of unemployment or potential unemployment used to estimate inflation at any quarter was a four quarter average of the past year. In some cases, expected inflation was reported on a biannual, rather than quarterly basis. In those cases, values such as the unemployment gap over unemployment, or regressors used in the spline regression were calculated at the quarterly level and then averaged into biannual data points. In the case of OECD data, potential unemployment was calculated on an annual basis. A value for potential unemployment to predict inflation for any quarter was an average of the previous four quarters, and it was presumed for OECD data each quarter had its year's value for NAIRU.

Median inflation was measured on a quarter over quarter basis; or, if necessary, a three month average of a month over month basis. Expected inflation was sourced from the US-based Survey of Professional Forecasters (“SPF”) which surveys market participants and asks them about their expectation for CPI and PCE inflation over the next 10 year. Expected inflation was also sourced from Consensus Economics' forecasts for Canada which gave 6-10 year estimates for Consumer price index (“CPI”) inflation in Canada. Additionally, expected inflation data was sourced from the International Monetary Fund (“IMF”) which polls member economists about their expectations of inflation in a country for the current year and the next five years. The estimates for inflation in the fifth year were taken as long-term inflation expectations.

III. Overall Strength of Phillips Curve Model

Table 1 shows the strength of short-term unemployment data in predicting median inflation less survey-measures of expected inflation. The adjusted R^2 values of the relations between inflation, less expected inflation and either the unemployment rate or the unemployment gap is shown on the next page. Generally, the strength of the regression ranges between strong ($R=.825$), to mediocre ($R=.16$). However, lower unemployment, measured either through the unemployment gap or rate, consistently predicts higher inflation, relative to expectations.

It seems that the Survey of Professional Forecasters for the Cleveland Fed gives the survey-based expectations for inflation that best fit into the augmented Phillips Curve. The strength of the Phillips Curve is middling, even for US data, when IMF surveys are used. The SPF surveys may be more useful for predicting inflation behavior because the survey-takers they consult are exclusively market participants. On the other hand, the IMF polls many economists who are not private-sector employed. Additionally, the long time frame for the SPF surveys may make them more useful as a tool to capture underlying inflation expectations.

IV. Forecasting Inflation from 2015-2019

To test whether the augmented Phillips Curve could have predicted to 2015-2019 data, I employed a strategy using a Hodrick-Prescott (HP Filter), as well as a strategy employing the CBO's Natural Rate of Unemployment estimate. The HP Filter strategy and its results will be covered in 3.1. The CBO strategy will be covered in 3.2 and the analysis will be covered in 3.3.

IV.A. Forecasting Inflation From 2015-2019 with A Hodrick-Prescott Filter

For the purposes of attempting to predict future inflation with a HP Filter, I presume one has access to unemployment data up to the end of 2019 to create a HP Filter data series, but only inflation data up until 2014. I use this inflation data to estimate parameters in an OLS regression of inflation less expectations against measures of labor market slack. Later labor market slack data and inflation expectations are then plugged into the estimated model (which presumes a 1-1 impact of expectations on inflation) to predict future inflation.

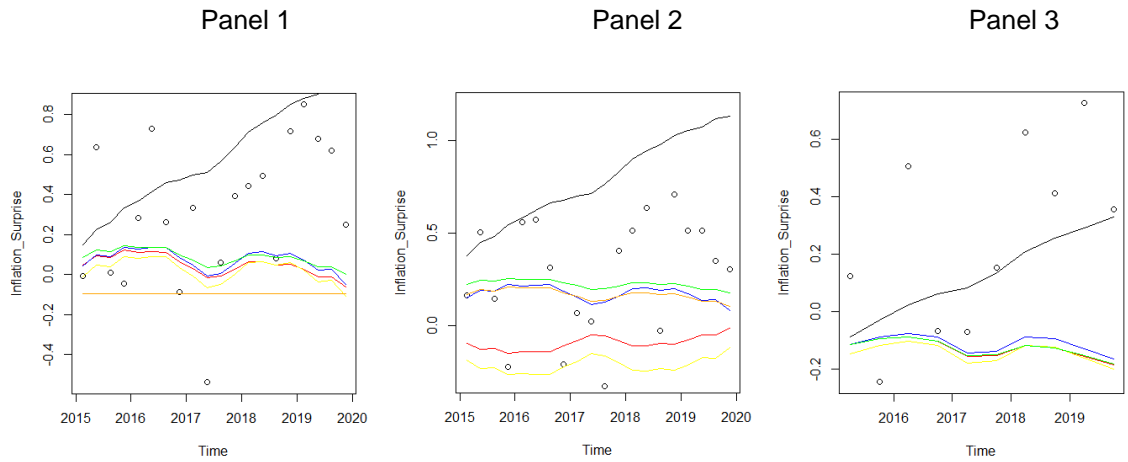
Panels 1-6 chart the attempts to predict inflation less expectations. The dots represent actual inflation. The lines represent modeled inflation. Black corresponds to inflation as a linear model of the short-term unemployment rate. Red corresponds to a linear short-term unemployment gap model. Yellow corresponds to the short-term unemployment gap over the short-term unemployment rate. Blue corresponds to the short-term unemployment gap over the short-term NAIRU. Orange corresponds to the cubic function. Green corresponds to the spline function.

Table 1

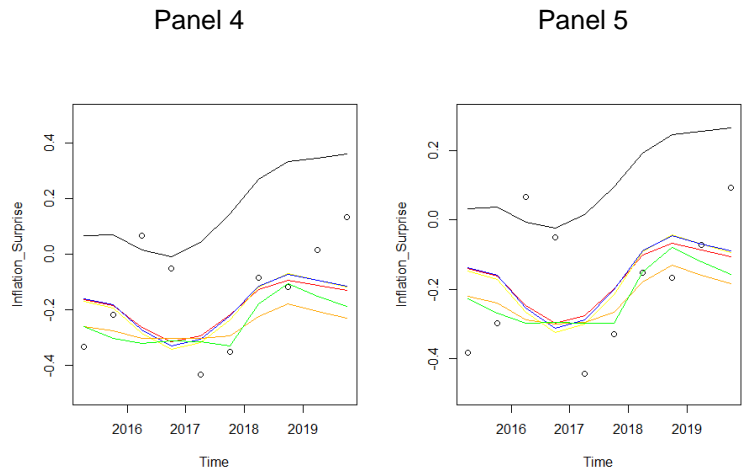
Country	PCE/CPI	Source for Exp.	Source for NAIRU	U (R)	U*-U (R)	Date Range/Data Points
USA	CPI	SPF	HPF	.5769	.5839	1991Q4-2019Q4 / 113
USA	CPI	SPF	US CBO	.5769	.5395	1991Q4-2019Q4 / 113
USA	CPI	SPF	OECD	.5785	.6051	1991Q4-2014Q4 / 93
USA	PCE	SPF	HPF	.5178	.5752	2007Q1-2019Q4 / 52
USA	PCE	SPF	US CBO	.5178	.541	2007Q1-2019Q4 / 52
USA	PCE	SPF	OECD	.6384	.657	2007Q1-2019Q4 / 32
USA	CPI	IMF	HPF	.2127	.2336	1995H1-2019H2 / 50
USA	CPI	IMF	US CBO	.2127	.0997	1995H1-2019H2 / 50
USA	CPI	IMF	OECD	.117	.2216	1995H1-2014H2 / 40
Canada	CPI	Consensus Economics	HPF	.2907	.1643	1995Q1-2019Q4 / 100
Canada	CPI	IMF	HPF	.1106	.1286	1995H1-2019H2 / 50
Canada	CPI	Consensus Economics	OECD	.3918	.0155	1995H1-2014H2 / 80
Canada	CPI	IMF	OECD	.1737	.0225	1995H1-2014H2 / 40
Australia	CPI	IMF	HPF	.1942	.0792	1992H1-2019H2 / 56
Australia	CPI	IMF	OECD	.5383	.0134	1992H1-2014H2 / 36

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USA - SPF Predicting CPI (Left) and PCE (Center) and IMF Predicting CPI (Right)



Canada - Consensus Economics Predicting CPI (Left) and IMF Predicting CPI (Right)



Australia - IMF Predicting CPI

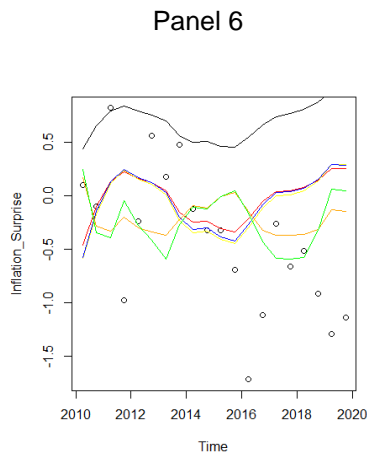


Table 2 shows the percent by which the variance of inflation is decreased by subtracting out modeled inflation (survey-based inflation expectation, plus model-predicted inflation surprise). A negative percent means that the modeled inflation is a worse model that projects inflation to remain constant; and that the variance of unexplained inflation has been increased.

Most values in the table are negative. Overall, a HP Filter-based approach to modeling a short-term Phillips Curve with survey-based inflation expectations failed to predict inflation behavior from 2014-2019. Using the employment gap over potential unemployment was consistently a superior means of predicting inflation than the unemployment gap over unemployment. Only SPF-collected CPI inflation expectations led to consistently more accurate modeling of inflation than the presumption it remains constant.

Table 2

	U	U*-U	(U*-U)/U	(U*-U)/U*	Cubic	Spline
USA - SPF (CPI)	10.7%	.2%	5.5%	5.4%	.6%	.7%
USA - SPF (PCE)	-10.2%	-2.7%	-7.2%	-6.1%	2.6%	2.5%
USA - IMF (CPI)	2.7%	-61.9%	-54.8%	-54.6%	-56.1%	-61.8%
Canada - Consensus Economics (CPI)	-4.8%	-6%	-7.4%	-6.9%	1.7%	6.1%
Canada - IMF (CPI)	-1.0%	-12.6%	-16.5%	-15.3%	-1.8%	-1.3%
Australia - IMF (CPI)	-34.3%	-35.7%	-47.1%	-43.3%	-23.3%	-70.5%

IV.B. Forecasting Inflation From 2015-2019 with CBO-Estimated NAIRU

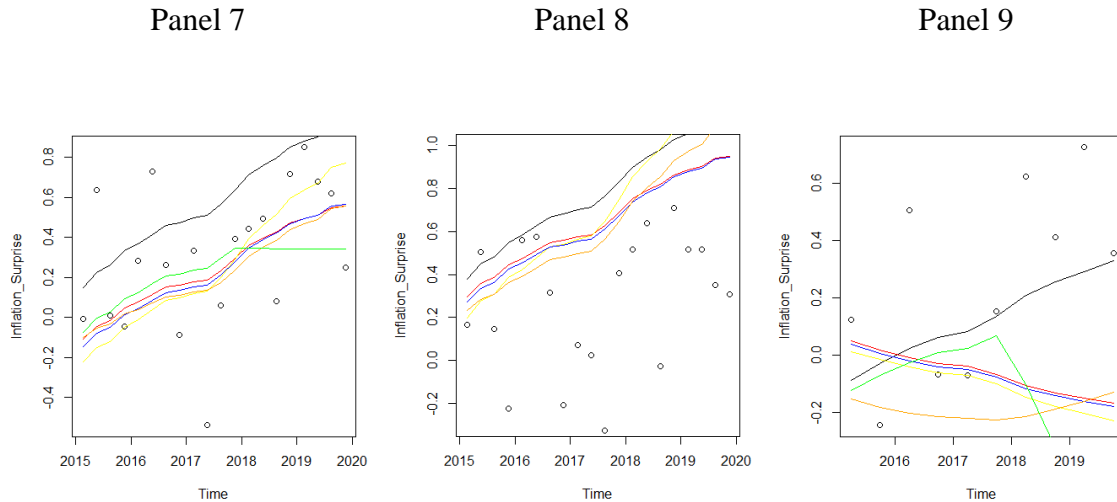
Previously, this paper used NAIRU estimates for total unemployment to estimate short-term NAIRU by finding the average value of short-term unemployment rates and total unemployment rates over a time-period considered and multiplying the total unemployment NAIRU by the short-term average and dividing by the total average. This strategy is applied again to turn CBO estimates of total NAIRU into a short-term NAIRU data series, but with averages that extend until the end of 2014, not the end of the time series. Parameters are found regressing inflation minus expectations against measures of labor market slack. Future labor market slack data and inflation expectations are used to predict future inflation. In the creation of these models, I assume that inflation expectations have a 1-1 direct effect on inflation.

All the data from this comes from the US. The results of these models are shown in panels 7-9. Again, the dots represent actual data, black the unemployment rate model, red the linear

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unemployment gap model, yellow the unemployment gap over the unemployment rate model, blue the unemployment gap over NAIRU model, orange the cubic unemployment gap model and green the spline gap model.

SPF estimated CPI (right), SPF estimated PCE (middle) and IMF estimated CPI (left):



In the case of the model based on SPF estimates of PCE. Within the time range 2007Q1-2014Q4, not once does unemployment rise above potential unemployment, so it is impossible to model a spline model in that case.

Table 3, shows the predictive power of the inflation model in percentage terms of the degree to which variance in unexplained inflation is reduced, in the same way table 2 showed the effectiveness of models for Section 3.1.

Table 3

	U	U*-U	$(U^*-U)/U$	$(U^*-U)/U^*$	Cubic	Spline
USA - SPF (CPI)	10.7%	17.3%	1.1%	16.1%	19.2%	9.8%
USA - SPF (PCE)	-10.2%	-1.6%	-45.9%	-2.8%	-21%	N/A
USA - IMF (CPI)	2.7%	-106.2%	-114.4%	-106.4%	-48.9%	-253.8%

Of the three inflation expectations series, the Survey of Professional Forecasters predictions of CPI seemed to be the only series that was useful in making predictions. While the two other

series largely gave worse predictions than assuming inflation would remain constant, this series consistently made better predictions.

Section 3.3: Analysis of the Results of Inflation Forecasts

Phillips Curve models estimated using inflation data up to the fourth quarter of 2014, failed to be useful when projected forwards to predict inflation data from 2015 to 2019. The one exception to this trend were models of CPI inflation which presumed a 1-1 relationship between CPI inflation and long-term CPI expectations as measured by the SPF.

Models involving CPI inflation and the SPF were formed out of a regression which involved more data points (93) than other models. They also covered the longest time period (1991Q4-2014Q4). Due to a lack of a counterfactual example, it is impossible to conclude that similar Phillips curve models with as many data points covering such a long time series could not have produced similarly strong forecasts as the SPF-based CPI forecasts. Nonetheless, there is no strong evidence that the success of this time series in making predictions is largely a fluke.

Hence, there is no reason to say that Phillips Curve models of the type this paper studied accurately predicted inflation behavior from 2015 to 2019.

V. Determining Concavity of Phillips Curve Through Comparing Regressions

To analyze the strength of various measures of labor market slack in influencing inflation, inflation is regressed against expectations and (a) measure(s) of labor market slack. Additionally, I regress inflation, less expectations against (a) measure(s) of labor market slack. The difference in the two regressions is whether one presumes that the effect of expectations on inflation is 1-1.

Regressions which model labor market slack as a spline function of unemployment include a regressor entitled “Spline Measure.” This regressor is equal to the unemployment gap (potential minus actual unemployment) when the unemployment gap is positive. It is equal to 0 when the unemployment gap is negative. One expects an increase in the unemployment gap (decline in unemployment) to increase inflation under a Phillips Curve model. If one expects concavity in the model, one should expect the effect of the unemployment gap to be more positive the closer unemployment gets to 0. Hence, one should expect the spline measure to have a positive parameter.

V.A. Determining Concavity Through Regressing Inflation:

Data on all regressions of inflation can be found in appendix #1. Those regressions, in the case where labor slack is understood as being a spline function of the unemployment gap, involve the regressor spline measure.

Table 4 shows that no method of measuring labor market slack in a regression of inflation yielded a consistently higher adjusted R^2 value. The inclusion of potential unemployment measures does not consistently increase the fit of the regressions.

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Generally, in Neo-Keynesian Phillips Curve (“NKPC”) models, a 1% increase in expected inflation is presumed to lead to a 1% increase in actual inflation (Roberts 1995). A T-test was performed against the null hypothesis that the slope parameter of expected inflation should equal 1. If an alpha value in such a test is less than .05, that is interpreted as a rejection of the null hypothesis that the expectation parameter is equal to 1. Such an outcome is assumed to represent a flaw in current understanding of the NKPC, or (more likely) a flaw in the regression, such as the labor market slack measure(s). In the chart in the next page are the results of regressions of inflation over unemployment and the unemployment gap. The values given are the estimated slope parameter for expectations, the alpha value for the t-test that the parameter equals 0, and the alpha value that the parameter equals 1. Data from regressions involving expectations from the IMF are highlighted in green.

Table 4

Country	PCE /CPI	Exp.	NAIRU	Unemployment Estimate / P for 0 / P for 1	Unemployment Gap Estimate / P for 0 / P for 1
USA	CPI	SPF	HPF	1.06 / p<.001 / p=.604	.78 / p<.001 / p=.021
USA	CPI	SPH	CBO	1.06 / p<.001 / p=.604	.74 / p<.001 / p=.01
USA	CPI	SPH	OECD	1.04 / p<.001 / p=.72	1.17 / p<.001 / p=.133
USA	PCE	SPF	HPF	1.16 / p=.087 / p=.807	-.01 / p=.983 / p=.068
USA	PCE	SPF	CBO	1.16 / p=.087 / p=.807	.99 / p=.126 / p=.983
USA	PCE	SPF	OECD	.51 / p=.495 / p=.511	.75 / p=.311 / p=.734
USA	CPI	IMF	HPF	.33 / p=.004 / p<.001	.235 / p=.018 / p<.001
USA	CPI	IMF	CBO	.33 / p=.004 / p<.001	.14 / p=.22 / p<.001
USA	CPI	IMF	OECD	.19 / p=.142 / p<.001	.3 / p=.013 / p<.001
Canada	CPI	C.E.	HPF	1.21 / p<.001 / p=.425	1.45 / p<.001 / p=.106
Canada	CPI	IMF	HPF	-.5 / p=.172 / p<.001	.406 / p=.004 / p=.07
Canada	CPI	C.E.	OECD	1.05 / p<.001 / p=.851	1.6 / p=.001 / p=.068
Canada	CPI	IMF	OECD	-.82 / p=.029 / p<.001	.49 / p=.268 / p=.239
Australia	CPI	IMF	HPF	1.2 / p=.013 / p=.666	.225 / p=.565 / p=.052
Australia	CPI	IMF	OECD	1.32 / p<.001 / p=.345	.11 / p=.78 / p=.025

Each regression over US data regarding expected inflation data from the IMF, passes the significance test (significance level of .05) demonstrating that inflation expectations do not equal 1. No regressions using non-IMF expectations data and unemployment as the measure of slack passed that significance test. Meanwhile, 5 out of 7 regressions involving unemployment and

IMF expectations data do. This implies that IMF collected inflation expectations are not as useful in constructing Phillips Curve models as other sources (in this case, the US SPF and Consensus Economics).

When one excludes regressions involving IMF-based expectations, one finds that generally unemployment alone as a measure of slack outperforms the unemployment gap in terms of the assumption that expected inflation should have a one-to-one impact on inflation. The parameter estimates are generally closer to one for regressions based purely on unemployment and not potential unemployment. This rule is broken only once, in the case of regressing inflation against SPF forecasts of PCE inflation and when the unemployment gap is measured with an adjusted OECD NAIRU estimate. Those two regressions have the least amount of data points for any regression (32) and cover the smallest span of years (4). The results they give are less reliable than all other results judging by the strength of large time periods and data sets. Hence, the evidence is strong that unemployment as a measure of labor market slack pairs better with the assumption of a one-to-one inflation and expectations relationship. Perhaps incorporating a measure of NAIRU into a Phillips Curve model may partially account for the effects of expected inflation on top of the inflation expectations parameter.

Table 5

Spline Measure Parameter Estimates / significance test p-values	HP Filter	OECD	CBO
SPF PCE US	-.377 (p=.203)	13.3 (p=.113)	-2.79 (p=.242)
SPF CPI US	-.504 (p=.058)	.0105 (p=.975)	-.49 (p=.248)
IMF CPI US	-.768 (p=.026)	-.626 (p=.267)	-.891 (p=.175)
C.E. CPI Canada	.828 (p=.001)	.323 (p=.364)	
IMF CPI Canada	1.04 (p=.011)	.666 (p=.294)	
IMF CPI Australia	.587 (p<.001)	-3.63 (p=.048)	

The spline regression would presumably yield evidence that the parameter associated with the “Spline Measure” is positive. If this were the case, a marginal decline in unemployment would generally create more inflation the lower unemployment is. This would fit in nicely with intuitions concerning the Phillips Curve. This is largely the conclusion found by Hooper, Miskin & Sufi (2020). The data, as shown in the chart on the next page is more complicated. As shown in that chart, of the 9 spline regressions involving US data, 7 involve a negative parameter associated with “Spline Measure.” A negative slope parameter for this implies that a marginal effect of lower unemployment on inflation is lesser when the economy is good. This is largely due to 2015-2019 data. 3 of the 9 regressions correspond to data sets which end in 2014. 2 of those 3 regressions involve a positive spline constant. All 4 spline regressions relevant to Canada

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deliver a positive spline parameter. Hence, it seems recent historical data justifies the possibility of a downward sloping, concave upwards Phillips Curve in Canada, but not in the US. Data from Australia is more mixed.

V.B. Determining Concavity Through Regressing Inflation less Expectations:

The Inflation-Augmented Phillips Curve employed by Ball & Mazumder (2018) (2015) presumed that a change in expected inflation would cause an equal change to inflation. They demonstrated the strength of their model by regressing inflation minus expected inflation against a measure of labor market slack (2015). This was done in all collections of time series gathered for this paper. All five measures of labor market slack, along with the short-term unemployment rate were used to explain inflation, less expectations. The estimated slope parameters and significance test alphas for each regressor, along with adjusted R^2 values are presented in the second appendix.

Once again, no measure of labor market slack, as measured by adjusted R^2 , is strictly more predictive than any other. Regressions in terms of the unemployment gap do not consistently match the data better than regressions in terms of unemployment.

Once again, 7 of 9 spline-based regression of US data finds a negative slope parameter, indicating a Phillips Curve which is concave down. This is shown in the chart on the next page. Additionally, each spline regression in Canada indicates a concave-upwards Phillips Curve, where rising unemployment has a declining marginal impact on inflation. Data from America indicates the opposite. It is hard to find a reasonable economic explanation for a concave-downwards Phillips Curve. However, data for the years 2014-2019 do show very little upward pressure on inflation at a time when unemployment fell and then remained very low.

Very likely, this atypical behavior of inflation needs a different explanation than the usual suspects in the Phillips Curve literature.

The reason this behavior is atypical is both because it violates intuition and has an important policy implication. A non-convex Phillips Curve is intuitive because there is a limit, even in the short term, on how low any unemployment rate can go (it can't go negative). Intuitively one would think that therefore, no matter how much unexpected inflation can be produced by demand side policies, it would past a certain point be unable to reduce unemployment without running into marginally declining effectiveness. Therefore, the marginal impact of lower unemployment in terms of necessitating higher inflation would be increasing implying a convex Phillips Curve.

Similarly, a non-provably convex Phillips Curve is baffling. Historically, the convexity of the Phillips Curve has been presumed and then used as an assumption to bolster the normative that economic booms are not worth the costs of their associated busts. The assumption was running inflation hot then cold (or cold then hot) in a way that stabilized inflation in the long run would produce more unemployment over time, on the net.

Table 6

Spline Measure Parameter Estimates / significance test p-values	HP Filter	OECD	CBO
SPF PCE US	-.512 (p=.069)	13.2 (p=.112)	-2.78 (p=.238)
SPF CPI US	-.511 (p=.06)	.0579 (p=.863)	-.437 (p=.315)
IMF CPI US	-.339 (p=.522)	-.356 (p=.654)	-.911 (p=.353)
C.E. CPI Canada	.876 (p=.001)	.401 (p=.262)	
IMF CPI Canada	.85 (p=.041)	.763 (p=.194)	
IMF CPI Australia	6.34 (p<.001)	-3.41 (p=.078)	

VI. Discussion

Ball and Mazumder's discovery of a statistically significant Phillips Curve effect in recent data using short-term unemployment and survey-based measures of inflation expectations is workable in other countries, with other data sources and is applicable to more recent data from the data sources they started with. This expansion of their discovery demonstrates that the New Keynesian Phillips Curve is not dead. However, the New Keynesian Phillips Curve, or attempts to measure it, still has/have limitations. There is little evidence to indicate that this expectations-augmented short-term unemployment would have had any predictive value, on its own, in estimating inflation behavior from 2015-2019.

Attempts to measure the shape of the Phillips Curve are largely inconclusive. Different applicable datasets appear to give different results in terms of indicating the structure of the Phillips Curve. Notably, data from America appears to indicate the inflationary impact of a marginal drop in unemployment is lower during periods of low unemployment. Meanwhile, data from Canada indicates the opposite. Six different methodologies were used to regress inflation and expectations data against labor market slack. Some methodologies fit better with some data, while others worked better with other data.

Attempts at finding a Phillips Curve using a normal unemployment rate will likely create stronger evidence for a concave-up Phillips Curve. Research has found that the apparent explanatory power short-term unemployment has over total unemployment declines when one uses a model which accounts for convexity (Speigner 2014). Hence, it is not surprising that the strong case for a nonlinear Phillips Curve found by other researchers (Sufi, Hooper & Miskin 2020) is not found in this paper.

It appears that IMF surveys were generally less useful in forecasting or explaining inflation than SPF and Consensus Economics Surveys. This effect could be due to a lack of vagueness in IMF

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surveys which asked participants about inflation expectations for a particular year. It could be due to IMF surveys operating on a shorter timescale (5 years vs. 10 years (SPF) or 6-10 years (C.E.)). The IMF surveys could have been less useful because they were more surveys of public economists rather than professional economists hired in the private sector. There could also be some other explanation.

VI.A. Policy Implications:

There is strong evidence of a Phillips Curve mechanism which allows inflation to be explained in terms of the behavior of unemployment. However, no strong evidence exists to suggest that inflation behavior can be predicted accurately in real time with a short-term expectations-augmented Phillips Curve. Surveys of market participants appeared more useful in explaining and forecasting inflation than a survey of more public economists by the IMF. This may be a spurious observation due to other differences between these surveys and a small sample size of surveys. Even if a useful model involving survey forecasts of inflation were developed, it would be important for Central Banks to not lean too heavily on such results. If private-sector economists, for instance, could influence interest rate policy by delivering a particular forecast, they may do so.

Additionally, no strong evidence exists to indicate that (in terms of short-term unemployment) the Phillips Curve is concave up. Hence, there is little reason to argue that pushing short-term unemployment 1% below potential is any more threatening than 1% above potential. This may not be true for usual unemployment measures.

VI.B. Implications for Future Research:

If it is the case that the Phillips Curve can better be demonstrated to be concave-up with total unemployment, then a formulation of a total unemployment-based concave-up Phillips Curve with a comparable predictive power to the one discovered by Ball and Mazumder may be possible. Likely, a good way to start would be by adopting median inflation indicators and survey-based expectation measures.

Alternatively, research could be conducted at a micro level to better understand labor market detachment. Analytical statistical work has been done to find that the longer someone is unemployed, the less likely that individual is to be able to get a job (Budd, Levine & Smith 1988) (Krueger et. al 2020). Many writers connect higher long-term unemployment to a shift in the Beveridge Curve and a higher NAIRU (Budd, Levine & Smith 1988) (Gordon 2013). Nonetheless an experiment in Sweden found no evidence of employer discrimination against job applications regarding length of unemployment (Eriksson, Stefan, and Rooth, 2014). Hence the explanation of this phenomenon might not stem from the employers. That said, employers in the countries studied in this paper may behave differently for cultural, regulatory or other reasons. Research into determining what factors, and to what extent, cause someone who is unemployed for a long period of time to not be as connected a member of the labor force compared to another worker is vital for work in both structural unemployment reduction and inflation targeting.

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VIII. Appendix #1: Regressing Inflation

Country: USA

NAIRU measure: HP Filter

Inflation Expectation Measure: Survey of Professional Forecasters (CPI)

Time Range: 1991Q4 - 2019Q4

Data Points: 113

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.06 p<.001	-.607 p<.001							p=.6041	.5429
$U^* - U$.779 p<.001		.924 p<.001						p=.0213	.5707
$\frac{U^* - U}{U}$.76 p<.001			4.28 p<.001					p=.0165	.5311
$\frac{U^* - U}{U^*}$.766 p<.001				4.35 p<.001				p=.0147	.5715
Cubic	.707 p<.001		.662 p<.001			-.105 p=.507	.216 p=.155		p=.0036	.5952
Spline	.781 p<.001		1.11 p<.001					-.504 p=.058	p=.0209	.5809

Country: USA

NAIRU measure: HP Filter

Inflation Expectation Measure: Survey of Professional Forecasters (PCE)

Time Range: 2007Q1 - 2019Q4

Data Points: 52

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.16 p=.087	-.419 p<.001							p=.8074	.4642
$U^* - U$	-.0119 p=.983		.684 p<.001						p=.0676	.5589
$\frac{U^* - U}{U}$	-.162 p=.774			3.27 p<.001					p=.0437	.5204
$\frac{U^* - U}{U^*}$.0665 p=.903				3.29 p<.001				p=.0908	.5627
Cubic	-.0207 p=.974		.52 p=.01			-.0541 p=.767	.106 p=.567		p=.1098	.5598
Spline	.252 p=.664		.826 p<.001					-.377 p=.203	p=.1996	.5648

The Phillips Curve

Country: USA

NAIRU measure: HP Filter

Inflation Expectation Measure: International Monetary Fund (CPI)

Time Range: 1995H1 - 2019H2

Data Points: 50

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	.328 p=.004	-.493 p<.001							p<.0001	.5146
$U^* - U$.235 p=.018		.894 p<.001						p<.0001	.6342
$\frac{U^* - U}{U}$.268 p<.001			3.94 p<.001					p<.0001	.5726
$\frac{U^* - U}{U^*}$.242 p=.016				4.11 p<.001				p<.0001	.6251
Cubic	.237 p<.001		.565 p=.005			-.119 p=.569	.273 p=.192		p<.0001	.6768
Spline	.202 p=.034		1.21 p<.001					-.768 p=.026	p<.0001	.665

Country: USA

NAIRU measure: Adjusted CBO estimate

Inflation Expectation Measure: Survey of Professional Forecasters (CPI)

Time Range: 1991Q4 - 2019Q4

Data Points: 113

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.06 p<.001	-.607 p<.001							p=.6041	.5429
$U^* - U$.742 p<.001		.577 p<.001						p=.01	.5306
$\frac{U^* - U}{U}$.7 p<.001			2.95 p<.001					p=.0036	.481
$\frac{U^* - U}{U^*}$.662 p<.001				2.14 p<.001				p=.0005	.5283
Cubic	.633 p<.001		.55 p=.001			.326 p=.11	.151 p=.018		p=.0004	.5682
Spline	.737 p<.001		.617 p<.001					-.49 p=.248	p=.0086	.5321

Country: USA
 NAIRU measure: Adjusted CBO estimate
 Inflation Expectation Measure: Survey of Professional Forecasters (PCE)
 Time Range: 2007Q1 - 2019Q4
 Data Points: 50

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.16 p=.087	-.419 p<.001							p=.8074	.4642
$U^* - U$.986 p=.126		.447 p<.001						p=.9828	.4893
$\frac{U^* - U}{U}$.766 p=.266			2.47 p<.001					p=.733	.4034
$\frac{U^* - U}{U^*}$.919 p=.153				1.48 p<.001				p=.8989	.4836
Cubic	.786 p=.206		.286 p=.459			.0999 p=.753	.0579 p=.413		p=.7291	.5342
Spline	.9565 p=.136		.47 p<.001					-2.79 p=.242	p=.9454	.4935

Country: USA
 NAIRU measure: Adjusted CBO estimate
 Inflation Expectation Measure: International Monetary Fund (CPI)
 Time Range: 1995H1 - 2019H2
 Data Points: 50

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	.328 p=.004	-.493 p<.001							p<.0001	.5146
$U^* - U$.139 p=.22		.555 p<.001						p<.0001	.5505
$\frac{U^* - U}{U}$.169 p=.167			2.71 p<.001					p<.0001	.4792
$\frac{U^* - U}{U^*}$.123 p=.282				2.04 p<.001				p<.0001	.5528
Cubic	.152 p=.158		.389 p=.105			.219 p=.476	.129 p=.163		p<.0001	.609
Spline	.14 p=.214		.622 p<.001					-.891 p=.175	p<.0001	.559

The Phillips Curve

Country: Canada

NAIRU measure: HP Filter

Inflation Expectation Measure: Consensus Economics (CPI)

Time Range: 1995Q1 - 2019Q4

Data Points: 100

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.21 p<.001	-.276 p<.001							p=.4253	.4181
$U^* - U$	1.45 p<.001		.38 p<.001						p=.1061	.3283
$\frac{U^* - U}{U}$	1.44 p<.001			2.48 p<.001					p=.1109	.352
$\frac{U^* - U}{U^*}$	1.46 p<.001				2.34 p<.001				p=.0969	.3344
Cubic	1.331p =.001		.242 p=.156			.497 p=.006	.343 p=.195		p=.2541	.3709
Spline	1.28 p=.001		-.0325 p=.825					.828 p=.001	p=.2984	.3942

Country: Canada

NAIRU measure: HP Filter

Inflation Expectation Measure: International Monetary Fund (CPI)

Time Range: 1995H1 - 2019H2

Data Points: 50

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	-.501 p=.172	-.3719 p<.001							p=.0001	.3128
$U^* - U$.357 p=.309		.406 p=.004						p=.0701	.1424
$\frac{U^* - U}{U}$.335 p=.332			2.64 p=.002					p=.0575	.1693
$\frac{U^* - U}{U^*}$.352 p=.315				2.47 p=.004				p=.0676	.1457
Cubic	.227 p=.508		.212 p=.418			.645 p=.029	.47 p=.311		p=.0278	.2006
Spline	.184 p=.584		-.116 p=.624					1.04 p=.011	p=.0182	.2395

Country: Australia
 NAIRU measure: HP Filter
 Inflation Expectation Measure: International Monetary Fund (CPI)
 Time Range: 1992H1 - 2019H2
 Data Points: 56

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.2 p=.013	-.55 p=.002							p=.666	.1449
$U^* - U$.225 p=.565		1.26 p=.012						p=.0515	.0877
$\frac{U^* - U}{U}$.21 p=.585			5.7 p=.004					p=.0433	.1217
$\frac{U^* - U}{U^*}$.209 p=.592				5.24 p=.009				p=.0459	.0971
Cubic	.314 p=.364		-.136 p=.889			8.45 p<.001	16.5 p=.066		p=.0508	.3425
Spline	.496 p=.172		-1.52 p=.09					5.87 p<.001	p=.1508	.2591

Country: United States
 NAIRU measure: Adjusted OECD Estimate
 Inflation Expectation Measure: Survey of Professional Forecasters (CPI)
 Time Range: 1991Q4 - 2014Q4
 Data Points: 93

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.04 p<.001	-.676 p<.001							p=.7195	.5835
$U^* - U$	1.17 p<.001		.853 p<.001						p=.1326	.619
$\frac{U^* - U}{U}$	1.15 p<.001			4.36 p<.001					p=.2207	.5934
$\frac{U^* - U}{U^*}$	1.19 p<.001				3.38 p<.001				p=.1115	.6191
Cubic	1.1 p<.001		.718 p<.001			.537 p=.065	.375 p=.026		p=.4077	.6347
Spline	1.17 p=.137		.85 p<.001					.0105 p=.975	p=.1374	.6147

The Phillips Curve

Country: United States

NAIRU measure: Adjusted OECD Estimate

Inflation Expectation Measure: Survey of Professional Forecasters (PCE)

Time Range: 2007Q1 - 2014Q4

Data Points: 32

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	.509 p=.495	-.607 p<.001							p=.5106	.6166
$U^* - U$.75 p=.311		.667 p<.001						p=.7337	.6322
$\frac{U^* - U}{U}$.829 p=.276			4.14 p<.001					p=.82	.615
$\frac{U^* - U}{U^*}$.782 p=.289				2.7 p<.001				p=.766	.6358
Cubic	.467 p=.55		1.19 p=.071			.73 p=.309	.225 p=.273		p=.4965	.6245
Spline	.697 p=.333		.615 p<.001					13.3 p=.113	p=.6719	.6523

Country: United States

NAIRU measure: Adjusted OECD Estimate

Inflation Expectation Measure: International Monetary Fund (CPI)

Time Range: 1995H1 - 2014H2

Data Points: 40

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	.188 p=.142	-.67 p<.001							p<.0001	.5833
$U^* - U$.298 p=.013		.784 p<.001						p<.0001	.6122
$\frac{U^* - U}{U}$.334 p=.009			3.74 p<.001					p<.0001	.5556
$\frac{U^* - U}{U^*}$.307 p=.011				3.43 p<.001				p<.0001	.6066
Cubic	.288 p=.012		.505 p=.008			.47 p=.346	.413 p=.128		p<.0001	.6604
Spline	.288 p=.016		.924 p<.001					-.626 p=.267	p<.0001	.615

Country: Canada
 NAIRU measure: Adjusted OECD Estimate
 Inflation Expectation Measure: Consensus Economics (CPI)
 Time Range: 1995H1- 2014H2
 Data Points: 40

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.05 p<.001	-.392 p<.001							p=.8514	.5058
$U^* - U$	1.6 p=.001		.167 p=.116						p=.0679	.2337
$\frac{U^* - U}{U}$	1.6 p=.001			1.34 p=.042					p=.0692	.2503
$\frac{U^* - U}{U^*}$	1.6 p=.001				1.27 p=.053				p=.0663	.2462
Cubic	1.54 p<.001		.0702 p=.733			.164 p=.468	.224 p=.544		p=.1177	.2195
Spline	1.57 p<.001		.0684 p=.973					.323 p=.364	p=.0901	.2321

Country: Canada
 NAIRU measure: Adjusted OECD Estimate
 Inflation Expectation Measure: International Monetary Fund (CPI)
 Time Range: 1995Q1- 2014Q4
 Data Points: 40

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	-.823 p=.029	-.576 p<.001							p<.0001	.4783
$U^* - U$.485 p=.268		.186 p=.295						p=.2392	-.0017
$\frac{U^* - U}{U}$.48 p=.262			1.47 p=.18					p=.2246	.0179
$\frac{U^* - U}{U^*}$.472 p=.272				1.35 p=.214				p=.2193	.0105
Cubic	.46 p=.307		-.0417 p=.904			.41 p=.305	.534 p=.433		p=.232	-.0173
Spline	.567 p=.201		-.135 p=.737					.666 p=.294	p=.327	.0062

The Phillips Curve

Country: Australia

NAIRU measure: Adjusted OECD Estimate

Inflation Expectation Measure: International Monetary Fund (CPI)

Time Range: 1992H1- 2014H2

Data Points: 46

	$E(\pi)$	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	$E(\pi)=1?$	Adj. R^2
U	1.32 p<.001	-.817 p<.001							p=.34047	.471
$U^* - U$.108 p=.78		.336 p=.368						p=.0253	-.0264
$\frac{U^* - U}{U}$.0581 p=.88			.798 p=.623					p=.0178	-.0402
$\frac{U^* - U}{U^*}$.057 p=.883				.729 p=.613				p=.0176	-.0399
Cubic	-.0618 p=.89		-.0745 p=.904			-3.92 p=.209	-3.85 p=.284		p=.0218	-.0346
Spline	.0706 p=.85		1.11 p=.039					-3.63 p=.048	p=.0165	.0429

Appendix #2: Regressing Inflation Less Expectations

Country: USA

NAIRU Measure: HP Filter

Inflation Expectation Measure: Survey of Professional Forecasters (CPI)

Time Range: 1991Q4 - 2019Q4

Data Points: 113

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-.591 p<.001							.5769
$U^* - U$.987 p<.001						.5839
$\frac{U^* - U}{U}$			4.61 p<.001					.5437
$\frac{U^* - U}{U^*}$				4.65 p<.001				.5822
Cubic		.852 p<.001			-.195 p=.225	.0615 p=.676		.5956
Spline		1.18 p<.001					-.511 p=.06	.5936

Country: USA

NAIRU Measure: Adjusted CBO Estimate

Inflation Expectation Measure: Survey of Professional Forecasters (CPI)

Time Range: 1991Q4 - 2019Q4

Data Points: 113

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-.591 p<.001							.5769
$U^* - U$.622 p<.001						.5395
$\frac{U^* - U}{U}$			3.21 p<.001					.4823
$\frac{U^* - U}{U^*}$				2.31 p<.001				.5156
Cubic		.614 p<.001			.208 p=.327	.0972 p=.133		.5513
Spline		.659 p<.001					-.437 p=.315	.5936

The Phillips Curve

Country: USA

NAIRU Measure: HP Filter

Inflation Expectation Measure: Survey of Professional Forecasters (PCE)

Time Range: 2007Q1 - 2019Q4

Data Points: 52

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-0.412 p<.001							.5178
$U^* - U$.719 p<.001						.5752
$\frac{U^* - U}{U}$			3.45 p<.001					.5312
$\frac{U^* - U}{U^*}$				3.46 p<.001				.583
Cubic		.655 p<.001			-.199 p=.221	-.0335 p=.840		.5823
Spline		.899 p<.001					-.512 p=.069	.6968

Country: USA

NAIRU Measure: Adjusted CBO estimate

Inflation Expectation Measure: Survey of Professional Forecasters (PCE)

Time Range: 2007Q1 - 2019Q4

Data Points: 52

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-0.412 p<.001							.5178
$U^* - U$.447 p<.001						.541
$\frac{U^* - U}{U}$			2.53 p<.001					.4624
$\frac{U^* - U}{U^*}$				1.49 p<.001				.5357
Cubic		.285 p=.456			.0872 p=.78	.0546 p=.431		.5806
Spline		.472 p<.001					-2.78 p=.238	.5449

Country: USA
 NAIRU Measure: HP Filter
 Inflation Expectation Measure: International Monetary Fund (CPI)
 Time Range: 1995H1 - 2019H2
 Data Points: 50

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-0.404 p<.001							.2127
$U^* - U$.652 p<.001						.2336
$\frac{U^* - U}{U}$			2.95 p<.001					.2108
$\frac{U^* - U}{U^*}$				3.03 p<.001				.2323
Cubic		.101 p=.727			.292 p=.359	.65 p=.044		.2832
Spline		.788 p=.005					-.339 p=.522	.2241

Country: USA
 NAIRU Measure: Adjusted CBO Estimate
 Inflation Expectation Measure: International Monetary Fund (CPI)
 Time Range: 1995H1 - 2019H2
 Data Points: 50

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-0.404 p<.001							.2127
$U^* - U$.29 p=.01						.0997
$\frac{U^* - U}{U}$			1.35 p=.036					.0691
$\frac{U^* - U}{U^*}$				1.01 p=.019				.0915
Cubic		.39 p=.288			.62 p=.188	.246 p=.083		.1594
Spline		.359 p=.011					-.911 p=.353	.0975

The Phillips Curve

Country: USA

NAIRU Measure: Adjusted OECD Estimate

Inflation Expectation Measure: Survey of Professional Forecasters (CPI)

Time Range: 1991Q4 - 2014Q4

Data Points: 93

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-.666 p<.001							.5785
$U^* - U$.794 p<.001						.6051
$\frac{U^* - U}{U}$			4.11 p<.001					.5821
$\frac{U^* - U}{U^*}$				3.54 p<.001				.604
Cubic		.683 p<.001			.604 p=.031	.415 p=.01		.628
Spline		.782 p<.001					.0579 p=.863	.6008

Country: USA

NAIRU Measure: Adjusted OECD Estimate

Inflation Expectation Measure: Survey of Professional Forecasters (PCE)

Time Range: 2007Q1 - 2014Q4

Data Points: 32

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-.616 p<.001							.6384
$U^* - U$.674 p<.001						.657
$\frac{U^* - U}{U}$			4.17 p<.001					.6418
$\frac{U^* - U}{U^*}$				2.72 p<.001				.6606
Cubic		.0112 p<.081			.604 p=.378	.186 p=.34		.6459
Spline		.624 p<.001					13.2 p=.112	.6752

Country: USA
 NAIRU Measure: Adjusted OECD Estimate
 Inflation Expectation Measure: International Monetary Fund (PCE)
 Time Range: 1995H1 - 2014H2
 Data Points: 40

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-0.36 p=.018							.117
$U^* - U$.568 p=.001						.2216
$\frac{U^* - U}{U}$			2.79 p=.002					.1986
$\frac{U^* - U}{U^*}$				2.52 p=.001				.224
Cubic		.406 p=.134			.872 p=.235	.573 p=.136		.2449
Spline		.646 p<.01					-.356 p=.654	.6008

Country: Canada
 NAIRU Measure: HP Filter
 Inflation Expectation Measure: Consensus Economics (CPI)
 Time Range: 1995Q1 - 2019Q4
 Data Points: 100

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-.283 p<.001							.2907
$U^* - U$.389 p<.001						.1643
$\frac{U^* - U}{U}$			2.53 p<.001					.1945
$\frac{U^* - U}{U^*}$				2.38 p<.001				.1706
Cubic		.237 p=.143			.532 p=.003	.37 p=.19		.2278
Spline		-.0514 p=.724					.876 p=.001	.2581

The Phillips Curve

Country: Canada

NAIRU Measure: HP Filter

Inflation Expectation Measure: IMF (CPI)

Time Range: 1995H1 - 2019H2

Data Points: 50

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-.206 p=.011							.1106
$U^* - U$.4 p=.006						.1286
$\frac{U^* - U}{U}$			2.57 p=.003					.1499
$\frac{U^* - U}{U^*}$				2.43 p=.002				.1308
Cubic		.256 p=.347			.535 p=.075	.354 p=.461		.1568
Spline		-.0291 p=.906					.85 p=.041	.1863

Country: Canada

NAIRU Measure: Adjusted OECD Estimate

Inflation Expectation Measure: Consensus Economics (CPI)

Time Range: 1995Q1 - 2014Q4

Data Points: 80

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-.395 p<.001							.3918
$U^* - U$.159 p=.321						.0155
$\frac{U^* - U}{U}$			1.32 p=.048					.0371
$\frac{U^* - U}{U^*}$				1.23 p=.064				.0311
Cubic		.787 p=.969			.244 p=.274	.351 p=.336		.0082
Spline		-.039 p=.85					.401 p=.262	.0189

Country: Canada
 NAIRU Measure: Adjusted OECD Estimate
 Inflation Expectation Measure: IMF (CPI)
 Time Range: 1995H1 - 2014H2
 Data Points: 40

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-0.311 p=.004							.1737
$U^* - U$.236 p=.176						.0225
$\frac{U^* - U}{U}$			1.71 p=.116					.0392
$\frac{U^* - U}{U^*}$				1.58 p=.142				.0311
Cubic		.0737 p=.825			.418 p=.257	.41 p=.497		.0053
Spline		-.142 p=.672					.763 p=.194	.0413

Country: Australia
 NAIRU Measure: HP Filter
 Inflation Expectation Measure: IMF (CPI)
 Time Range: 1992H1 - 2019H2
 Data Points: 56

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-.507 p<.001							.1942
$U^* - U$		1.2 p=.02						.0792
$\frac{U^* - U}{U}$			5.41 p=.008					.1086
$\frac{U^* - U}{U^*}$				4.91 p=.016				.0854
Cubic		.213 p=.829			8.91 p<.001	12.6 p=.158		.3347
Spline		-1.79 p=.045					6.34 p<.001	.2781

The Phillips Curve

Country: Australia

NAIRU Measure: Adjusted OECD Estimate

Inflation Expectation Measure: IMF (CPI)

Time Range: 1992H1 - 2014H2

Data Points: 46

	U	$U^* - U$	$\frac{U^* - U}{U}$	$\frac{U^* - U}{U^*}$	$(U^* - U)^2$	$(U^* - U)^3$	Spline measure	Adj. R^2
U	-0.747 p<.001							.5383
$U^* - U$.484 p=.211						.0134
$\frac{U^* - U}{U}$			1.02 p=.552					-.0144
$\frac{U^* - U}{U^*}$.894 p=.557				-.0146
Cubic		-.0976 p=.881			-1.13 p=.708	.118 p=.972		-.0037
Spline		1.21 p=.033					-3.41 p=.078	.0615