Unemployment and Okun's Law in the United States, 1948-2005

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The torpid recovery of the job market in the wake of the 2001 recession has continued to be the focus of much discussion among economists and policy makers as job growth continues to only creep upwards from its low four years after the start of the recovery. Correspondingly, unemployment numbers remained seemingly high for much longer than forecasters had predicted, even as the rest of the economy continued to gather speed. Anecdotally, this behavior has been attributed to high productivity growth during the '90's, growth which enabled firms to meet recovering demand after the recession without hiring new workers. However, only a statistical analysis of the behavior of unemployment—historical and recent—can reveal to what extent this behavior really does differ from historical norms, and to what extent it can be attributed to productivity gains.

This paper will follow the approach taken by Robert Gordon in a 1984 study that poses almost exactly these questions. In his case, however, the provocation was not unusually slow employment growth, but the unusually fast employment growth during the 1983-84 recovery. Gordon's approach is to link unemployment with GNP via the estimation of an Okun's Law coefficient, which, more accurately speaking, links the unemployment and output "gaps." (Since this paper will make use of GDP—as opposed to GNP—data, the following description of the proposed model will also use that term.) The 'GDP gap' refers to the "percentage difference between *actual* and *potential* real [GDP]," (Gordon, 1984) potential GDP being the level of output the economy *could* produce under conditions of full employment (Okun, 1970). Likewise the unemployment 'gap' can be understood as the difference between the actual unemployment rate and the 'full-employment' unemployment rate. Okun's Law states that the "unemployment gap is a constant fraction, *k*, of the output gap", approximately equal to 0.3 (Gordon, 1984). The recent recovery's seemingly slow decrease in unemployment implies a similarly slow decrease in the unemployment gap, and therefore—via Okun's law—a correspondingly slow decrease in the output gap. This paper will examine the extent to which the recent behavior of unemployment differs from historical norms—or perhaps the extent to which it has altered historical norms.

This analysis, after Gordon's original, will begin with a simple identity that relates real GDP (Q) with the unemployment rate (E/L), hours per employee (H), labor productivity (Q/EH), the labor force participation rate (L/N), and population (N) (539).

(1) $$Q \equiv (E/L)(Q/EH)(L/N)\,H\,N$$

Next, each of the components on the right side of the identity will be detrended, and individually related to detrended real GDP. The statistical estimate of the relationship between the employment rate and real GDP will be a "historical Okun's law relation" that can be used to estimate the (unobservable) level of potential GDP.

Usefully, Gordon's paper also provides the framework for decomposing potential GDP *growth* into "the growth rates of population, labor force participation, hours per person, and productivity per hour," complicating factors that Okun circumvented in his original paper by assuming that the magnitudes of each of their effects would show up in the unemployment rate (Okun, 1970). Gordon's approach, however, allows the estimation of the individual magnitudes of these additional factors, an exercise which will reveal the extent to which productivity gains have actually been responsible for slow employment growth.

I. Okun's Law and the Output Identity

The regular relationship between the GDP gap and the unemployment gap that was popularized by Okun's original work is typically thought to be between 2.5 and 3.0. The remaining variation in the GDP gap, however, can be attributed to the remaining components of identity 1: productivity, labor force participation, average hours per employee, and population. In order to estimate these relationships, however, identity 1 must be complicated slightly. Whereas employment, participation rate, and population data are measured from the entire civilian population, productivity and hours data are private non-farm business data. Additionally, employment data differ between the civilian sector and the non-farm business sector. Gordon's solution is to add two "mixed effects" variables to the original identity. In the equation that follows, the NF notation denotes those variables for the non-farm business sector. Those without are for the entire civilian labor force.

(2) $$Q \equiv (E/L)(Q/EH)(L/N)\, H\, N\, (Q/Q^{NF})\, (E^{NF}/E)$$

The purpose of these variables is to control for the effects of the mixed data set: each variable changes "whenever there is a change in the ratio of total output per civilian employee, Q/E, to the same ratio" in the non-farm business sector (Gordon, 1984). Finally, Gordon simplifies this identity by replacing the ratios with single letters: R stands for the employment rate, Z for productivity, F for the labor participation rate, M^q for the output mix variable, and M^e for the employment mix variable.

(3) $$Q \equiv R\, Z\, F\, H\, N\, M^q\, M^e$$

This equation and two variations on it will be used throughout the rest of this paper. The first variation is simply the identity for the *growth rate* of real GDP, in which each variable is expressed in terms of its annual growth rate (Gordon, 1984):

(4) $$q \equiv r\, z\, f\, h\, n\, m^q\, m^e$$

The second variation to be used in this study is more complicated: it transforms each variable into the natural log of the ratio of that variable to its own trend. For example, the GDP data is transformed into a new variable, Q' via the following calculation: Q' = ln(Q/Q*), where Q* is a trend variable. The second variation on identity 3, with each variable transformed, is shown below (Gordon, 1984).

(5) $$Q' \equiv R'\, Z'\, F'\, H'\, N'\, M^{q}{}'\, M^{e}{}'$$

Essentially, this equational form approximates each variable's "gap" from its full employment rate—such as the GDP gap and unemployment gap already discussed—in terms of that variable's deviation from trend. Identity 5 states that "deviations from trend in the employment rate, [R'], productivity, [Z'] and the other components *must sum to the deviation from trend of real [GDP, Q']*", *which in turn is the GDP gap* (Gordon, 1984. Author's emphasis). In this way, it is possible to target for statistical analysis those variations that cause—and are caused by—movement of each component of the output identity relative to its “natural,” “full employment” level.

In the context of equation 5, then, Okun’s law states that the employment gap approximated by R' is a constant fraction, k, of our approximated output gap, Q':

$$k = R'/Q' \qquad (6)$$

By extension, Gordon points out, it is implied that $1 - k$ must be equal to the sum of the ratios of the remaining variables to the output gap:

$$1 - k = Z' + F' + H' + N' + M^q + M^e \qquad (7)$$

In the regression framework that will be described shortly, the relationship between the output gap and each of the other components of the identity will be estimated individually. As dictated by equations 6 and 7, the estimated coefficients for all the components on the right side of equation 5 add up to the total effect—on the output gap—of a change in those components. To evaluate each component’s contribution to the total effect, the ratio of its estimated coefficient to the sum total of the coefficients is calculated. The resulting number for the employment gap variable will be an Okun coefficient.

II. Regression Model Specification of the Output Identity

The second variation of the output identity, shown above as equation 5, is the one adapted for regression analysis. There are several steps to the adaptation process, which will be described in detail below. I will first describe the data in its original form, and then the process by which each variable is transformed into the form required by equation 5 (that is, the natural log of the ratio of each variable to its trend).

A. The Data

The data for real GDP (Q) was obtained from the National Income and Product Accounts Tables published by the Bureau of Economic Analysis, and is measured in billions of chained (2000) dollars. It and all other data to be described are measured quarterly from 1948:1 to 2005:2.

In order to construct the output mix variable, real non-farm business output was also obtained from the NIPA tables. It is also measured in billions of chained (2000) dollars. The variable M^q was constructed by dividing real non-farm business output into real total GDP.

Labor force participation rate data (F), hours data (H), and population data (N) were all obtained from the Bureau of Labor Statistics. The labor force participation rate is measured for the entire civilian labor force. Hours are measured in thousands of hours for the non-farm business sector, but are transformed into average hours per person by dividing non-farm employment data into total hours. Population data measure the civilian non-institutional population, age 16 and over, in thousands of persons.

Productivity data (Z) were constructed using the non-farm business output data and the non-farm business hours data described above. Non-farm business output was transformed from billions of dollars to actual dollars and then divided by hours of all persons—which was likewise transformed from thousands of hours to actual hours—to get a measure of output per hour, or productivity, in real dollars per hour.

The employment rate variable (*R*) was constructed using employment level data and labor force data, both measured in thousands for the entire civilian labor force, and both obtained from the BLS. Employment level data were divided by labor force data to calculate the employment rate.

The employment mix variable (M^e) was constructed using the employment level data described above and data for non-farm business employment, measured in thousands, also obtained from the BLS. Non-farm business employment data were divided by total civilian employment data in order to create the M^e variable.

Finally, unemployment rate data were obtained from the BLS for the entire civilian population, aged 16 and over. Although the unemployment rate is not a part of the output identity as described above, and it is not a variable that will be included in the regression analysis, it will be used in the calculations to be described in the following section.

B. Model Specification

The form of equation 5 requires that each variable be in the form of the natural log of the ratio of that variable to its trend. Two different approaches were tested in estimating trends for each of the variables described above: a slight modification of Gordon's original approach, and a further modification to be described shortly. Although the second modification of Gordon's approach is ultimately chosen, describing the first approach is a useful exercise because it yields an interesting byproduct: an estimation of the growth rate of potential GDP over the period from 1948 to 2001.

Estimating Trend Series: Gordon's Original Approach

In the original approach, Gordon begins with the estimation of a benchmark unemployment rate—that is, determining the unemployment rate that at a given time represents "full" employment. Recall that it is only when the actual unemployment rate equals the natural unemployment rate that actual real GDP equals potential GDP. For this benchmark unemployment rate, Gordon uses a series he estimated in a previous study, in which he estimates the natural unemployment rate by relating it to, among other things, the rate of inflation (Gordon, 1982).

The actual unemployment rate passes through the natural unemployment rate twice during each business cycle—once as it is rising at the end of an expansion, and once when it is falling during a recovery period. Gordon chooses the second point in each cycle as a marker for the end of that cycle and the beginning of the next (Gordon, 1984). The dates of these cycles do not exactly match those of the business cycle established by the National Bureau of Economic Research, due to the varying lag of the unemployment rate's response to changes in output. However, the unemployment rate's cycle is the more relevant here, because it is the specific relationship between unemployment and output that we are interested in.

The intention of establishing these benchmark periods is to capture the very different behavior of such variables as productivity, hours, and participation during each cycle of the unemployment rate. Gordon estimates separate trends for each benchmark period for each component of the output identity, and uses these piecemeal trend series to construct the variables for equation 5.

Estimating Trend Series: First Modification

Gordon's original approach is slightly problematic because the estimated series he uses for the natural unemployment rate is based in part on the relationship between unemployment and inflation, a theoretical relationship that has fallen out of favor in recent years due to its poor empirical track record, (Espinoza-Vega and Russell, 1997). In general, estimating the natural unemployment rate is a difficult problem because this simple Phillips curve method is unavailable (Clark). Furthermore, while there has been "considerable theoretical progress" in recent years regarding the determinants of the natural unemployment rate, empirical research has not matched this progress. According to Olivier Blanchard and Lawrence Katz in their working paper entitled *What We Know and Do Not Know About the Natural Rate of Unemployment,* "Economists do not have a good quantitative understanding of the determinants of the natural rate, either across time or across countries," (1996).

This certainly poses a problem in re-creating Gordon's approach. However, Gordon himself provides the answer within the very same study: the transformation of the variables for the output identity in equation 5 approximates each variable's "gap" as that variable's deviation from its own trend. In other words, the estimated trend of GDP stands in for potential GDP; the estimated trend of the employment rate stands in for the natural employment rate, and so on. If this technique produces an acceptable stand-in for the natural *em*ployment rate, the same technique should acceptably approximate the natural *un*employment rate, and without relying on the theoretically suspect Phillips curve approach. Therefore, in the first modification of Gordon's original approach, the benchmark periods were established using a trend series of the actual unemployment rate as a stand-in for the natural unemployment rate. More precisely, the quarter chosen as the benchmark for each cycle was the quarter preceding the quarter where the actual unemployment rate was closest to the estimated natural rate, which was done to allow for the lagged response of the unemployment rate to changes in GDP. Using this technique, the benchmark quarters were determined to be the following: 1948:2, 1953:3, 1957:3, 1960:1, 1970:4, 1974:2, 1979:4, 1990:3, and 2001:3. Curiously, they almost exactly match Gordon's benchmark quarters for the years that this study overlaps his, rarely differing by more than one quarter.[1] Figure 1 on the plots the actual unemployment rate and its trend, showing the fluctuation of the actual rate around our estimated natural rate. Table 1 shows the calculated growth rates of each component of the output identity.

Essentially, the data in Table 1 is in the form of the first variation of the output identity. According to equation 4, the growth rates of each component of the output identity must sum to the growth rate of real GDP. However, the division of the data is such that the growth rates are measured from one period of "full employment" to another. Therefore, the sum of the growth rates of the output identity provide an estimate of the growth rate of *potential* GDP over each benchmark period. The shaded column of Table 1 displays these estimates. For comparison's sake, the final column shows the observed growth rate of actual real GDP.

On the whole, these calculations display the same trends as Gordon's originals, although the potential growth estimates are consistently higher in this paper's than in the original. The rapid potential growth of the Korean War cycle (1948:2-1953:3) is

Figure 1: Unemployment Rate and Estimates Trend, 1948-2005

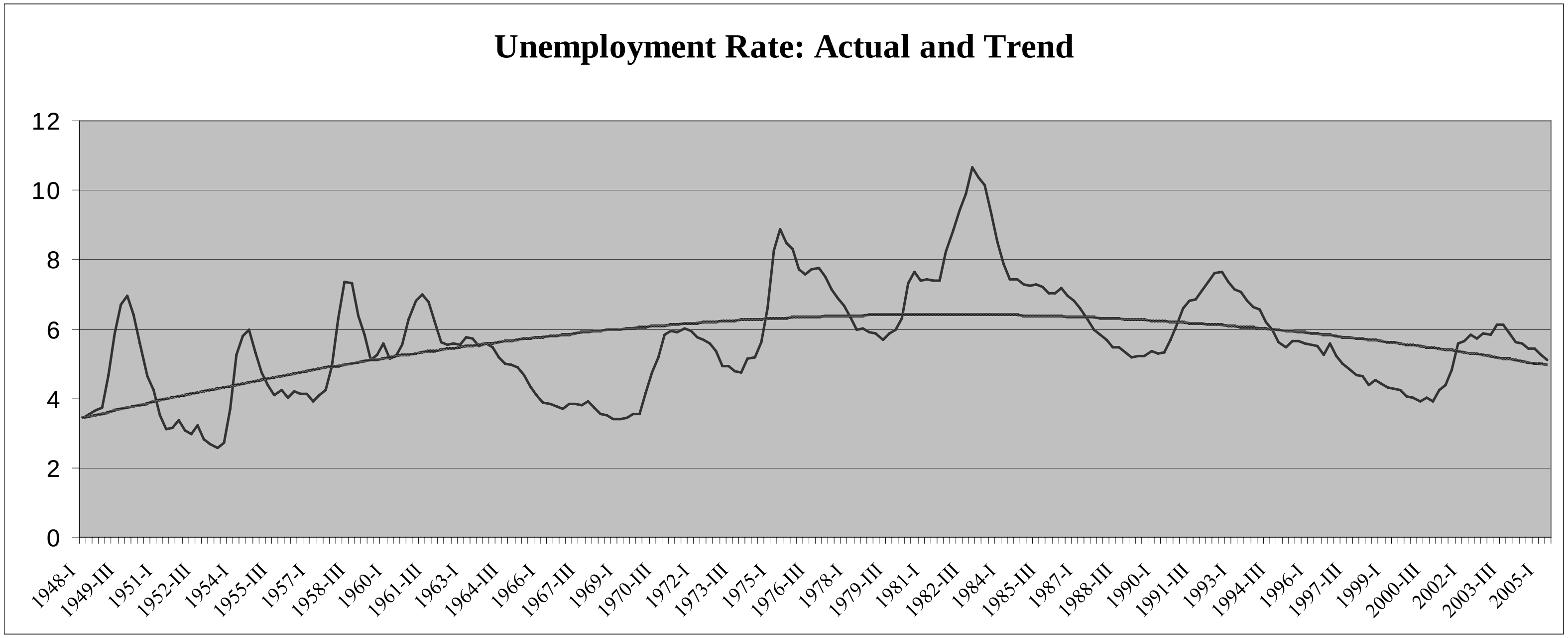


Table 1: Growth Rates of Potential real GDP and Components of Identity between Benchmarks, 1948-2005

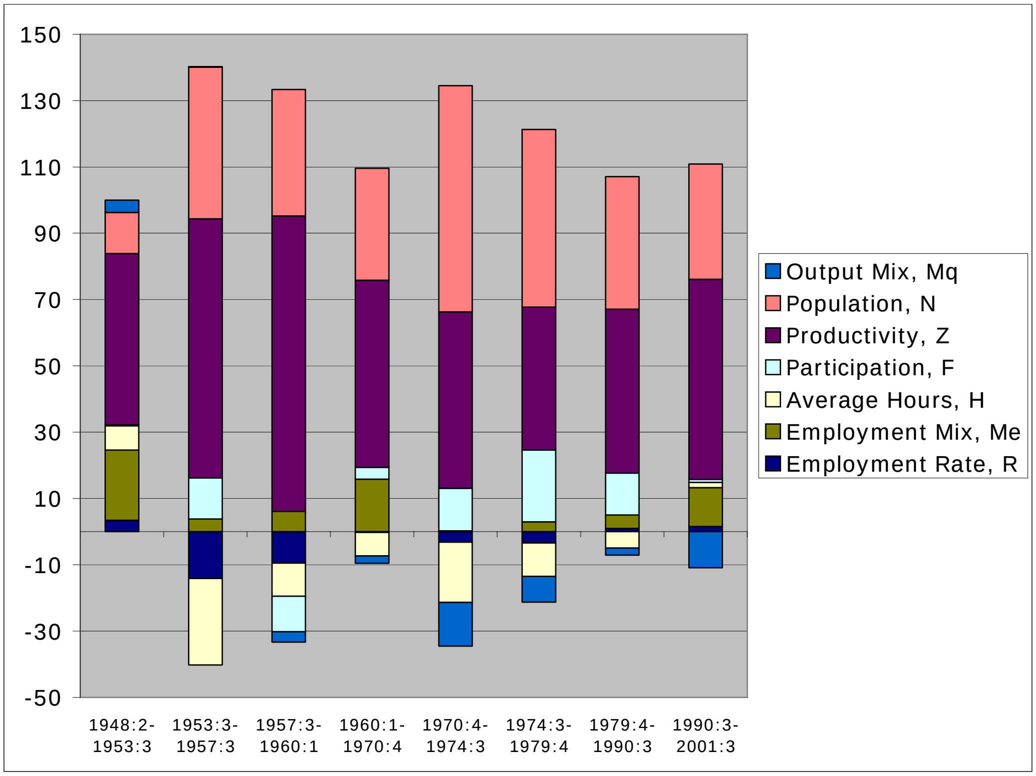
Period	*Employment Rate, r*	*Productivity, z*	*Participation Rate, f*	*Average Hours, h*	*Population, n*	*Output Mix, m^q*	*Employment mix, m^e*	*Potential Real GDP, (estimated) $q_{potential}$*	*Real GDP (observed) q*
1948:2-1953:3	0.214	3.228	0.017	0.459	0.778	0.231	1.323	6.252	4.920
1953:3-1957:3	-0.368	2.034	0.322	-0.679	1.193	0.002	0.099	2.604	2.458
1957:3-1960:1	-0.372	3.495	-0.418	-0.391	1.494	-0.124	0.237	3.919	3.633
1960:1-1970:4	-0.012	2.702	0.171	-0.337	1.616	-0.108	0.757	4.789	3.990
1970:4-1974:3	-0.107	1.786	0.431	-0.610	2.293	-0.441	0.007	3.360	3.286
1974:3-1979:4	-0.121	1.527	0.765	-0.357	1.897	-0.274	0.104	3.542	3.422
1979:4-1990:3	0.030	1.545	0.396	-0.156	1.249	-0.064	0.124	3.124	3.001
1990:3-2001:3	0.052	2.044	0.030	0.054	1.176	-0.368	0.396	3.385	2.959
2001:3-2005:2	-0.398	4.297	-0.424	-1.346	1.265	-0.464	-0.697	--------	2.972

estimated here at approximately 6.3%. The remainder of the 50's displays a potential growth rate between 2.6% and 3.9%, and potential growth in the 60's rises to approximately 4.8%. The 70's, 80's, and 90's display only a slight variation in potential growth rate, ranging from 3.1% to 3.5%. The period from 2001:3 to 2005:2 is included so that the behavior of each variable can be observed, but because the cycle of which it is a part has not yet come to a close, the growth of potential GDP cannot be inferred from its data.

Other notable behavior is displayed by the data in this table. In the first column, the growth rate of the employment rate is negative for the benchmark periods between 1953 and 1979, which is consistent with Gordon's calculations. He pointed out that this persistent decline was a reflection of the corresponding *in*cline of the natural unemployment rate over time, an incline that exists in both his and my estimates (Gordon, 1984). During the 80's and 90's, however, the implication is that the natural rate of unemployment fell. The second column displays productivity growth rates for each benchmark period. The Korean War cycle again displays rapid productivity growth, followed in the next three periods by growth rates ranging from approximately 2.0% to 3.5%. The productivity growth slow-down of the 1970's is displayed in the next two benchmark periods—averaging 1.8% and 1.5% growth rates. No amelioration of the slowdown is evident in during the 80's, but an increase to approximately 2.0% during the 90's could signal a change in the trend. This seems even more likely when the 4.3% productivity growth from 2001:3 to 2005:2 is noted (although its cyclical contamination must also be taken into account). Finally, the last notable phenomenon observable in Table 1 is the sharp increase in the labor force participation rate during the 70's, which can be largely attributed to the entrance of women into the workforce in larger numbers (Gordon, 1984). The subsequent drop-off in the growth rate of labor force participation since then indicates that the effects of this phenomenon have long since leveled off.

Figure 2 below shows the contribution of each component of the output identity to the estimated growth rate of potential GDP. The average growth rate for each variable over each benchmark period is calculated as a fraction of the potential growth rate. In this form, the large percentage of potential growth contributed by productivity growth can be appreciated. Notable is the steady climb of its share of the total since its drop-off in the 1970's.

Figure 2: Contributions to Potential GDP Growth Rate



Estimating Trend Series: Second Modification

Having detoured slightly—although usefully—from the description of the regression model's specification, it is worth re-stating here that the first modification involved estimating the natural unemployment rate using its own trend, dividing the data into benchmark periods, and estimating individual trends for each variable in each period, which trends are then used to construct the variables for equation 5. The second modification, however, estimates trends for each variable over the entire period from 1948:1 to 2005:2, and then uses those trends in creating the variables for equation 5. Trend models were estimated using both techniques, and trend data generated via in-sample forecasts of both models. The error statistics of each technique were compared (specifically the Mean Squared Error and Mean Absolute Deviation) and the better trend model chosen based on these. The second modification—in which trends were estimated for the whole period as opposed to in individual benchmark periods—consistently provided the better estimates according to the error statistics mentioned above. Therefore, the variables for equation 5 were constructed using trend data generated by an in-sample forecast of trend models estimated over the period from 1948:1 to 2005:2. Appendix A contains the regression output for each trend model of the second modification, as well as a graph of the actual and fitted data. These are included as an appendix for thoroughness' sake, but will not be discussed further.

III. The Regression Model

In order to interpret the recent behavior of unemployment, it is necessary to (finally) identify its *context,* that is the normal cyclical patterns linking the components of the output identity. Equation 5 "decomposes the detrended output ratio, [Q'], into the detrended values of the other components of the output identity," (Gordon, 1984). In order to allocate the "observed [GDP] gap among the other components of the identity…we can express each component as a linear function of current and lagged values" of the GDP gap (Gordon, 1984):

$$Y' = a_i + \sum_{s} b_{is} Q'_{t-s} + u_{it} \tag{8}$$

Where Y' stands for—by turns—each of the seven components of the output identity (*R', Z', F', H', N',* M^q*', and* M^e*'*). The adding-up constraint that originates with equation 5—the necessity that the deviations in trend of the seven components collectively sum to the deviation from trend of GDP—is retained in equation 8. More specifically, the "adding-up conditions" require that $\sum \alpha_i = 0$, $\sum \beta_{i0} = 1$, and $\sum_I \beta_{is} = 0$ for all $s \neq 0$ (544).

After Gordon's original, equation 8 was estimated for each component of the output identity with the current value and four lags on the log output ratio. Also similar to Gordon's original, significant serial correlation was present in these equations, which was ameliorated by the addition of four lagged variables of the dependent variable. The final equation for all seven of the dependent variables to be regressed, then, was as follows:

$$Y'_{it} = a_i + \sum_{S=1}^{4} c_{is} Y'_{i,t-s} + \sum_{S=0}^{4} b_{is} Q'_{t-s} + u_{it} \tag{9}$$

where Y' still stands for each of the seven components of the output identity by turn. The disadvantage of this approach is that the adding-up conditions imposed by equation 8 are lost due to the addition of the lagged dependent variables, a problem that will be discussed further in the following section.

IV. Results

The seven regression equations were estimated for two separate time periods: the first was for the period from 1948:4 to 1979:3, the same time period as Gordon's original study. By comparing the two sets of results, it can be known whether any changes observed in later time periods are solely the result of changes in the behavior of the variables, or if they can also be attributed to differences between Gordon's model and mine. The second set of equations was estimated for the entire period from 1948:2 to 2001:3 in order to assess how the long term historical relationships between the components of the output identity may have changed since 1979. Again, the cycle from 2001:3 to 2005:2 has not yet come to an end, rendering that data cyclically contaminated and inappropriate for regression here.

The regression results for the time period 1948:4-1979:3 are contained in Table 3. However, the elements that are important for a comparison of Gordon's results and mine are contained in Table 2. In it, the long run effects of a change in the output ratio on each of the seven dependent variables as estimated by Gordon and as estimated in this study are compared.

The primary difference is in the estimated "Okun coefficient," which Gordon pegs at 0.492, and which this study estimates to be 0.308—a number consistent with the accepted historical norm, which Gordon's was not. Other differences include a slightly greater long run effect on productivity; a slightly lesser long run effect on the participation rate; and an opposite sign on the coefficient for average hours. The differences between Gordon's results and mine for the same time period can be attributed to a number of factors, among them the different specification of trends for the construction of each variable, and using GDP as opposed to GNP data. Whatever their cause, however, these differences are significant enough to render comparison of Gordon's results for this time period and mine for later time periods impractical. Therefore, the analysis will concentrate on the differences between my results for this time period and my results for the 1948-2001 time period.

Table 2: Comparing Results

Variable	***Gordon's Original Estimates*** *Long run effect of a change in Q'**	***This Study's Estimates*** *Long run effect of a change in Q'**
Employment Rate	0.492	0.308
Productivity	0.014	0.055
Participation	0.155	0.055
Average Hours	-0.353	0.083
Population	0.008	0.058
Output Mix	-0.167	-0.041
Employment Mix	0.449	0.048

*Each number is the ratio of the sum of that variable's *Q'* coefficients to the sum of all seven variables' *Q'* coefficients.

Tables 3 and 4 on the following pages shows the complete results of the regressions for 1948:4-2001:3 and 1948:2-2001:3. The long run responses of Y' to changes in Q' over both time periods—what we are really interested in—are shown in the last line of each table. For comparison, those numbers are reproduced alone in Table 5.

The original intent of this study was to target and evaluate the recent behavior of employment in the context of its historical norms, and also to assess the anecdotal assertions that its recent, seemingly slow, recovery after the 2001 recession can be attributed to the productivity gains of the 90's. The regression results summarized in Table 5 have implications on both fronts. Between the first and second sets of regressions the estimated Okun coefficient drops from 0.3 to 0.2. The Okun's Law relationship of approximately 3-to-1 between output and unemployment has been previously determined to be sufficiently stable to deserve being called a law. However, these results suggest that the relationship between the output and employment gaps (and between the output and corresponding *un*employment gaps) varies over time.

Table 3: Regression Results for Components of GDP Identity, 1948:4-1979:3

Independent Variable	Lag length, s	Employment Rate, r	Product-ivity, z	Participation rate, f	Average hours, h	Population, n	Output mix, m^q	Employment mix, m^e
Constant		-0.000	0.000	-0.000	-0.000	-0.000	5.38E-05	-0.000
Lagged dependent variable	1	1.256**	1.078**	0.953**	0.878**	1.439**	0.729**	0.864**
	2	-0.633**	-0.037	-0.026	-0.055	-0.276	0.246*	-0.011
	3	0.229	-0.190	0.011	0.101	-0.088	-0.097	0.077
	4	0.040	0.106	0.039	0.048	-0.078	0.028	-0.053
Output ratio, Q’	0	0.200**	0.714**	-0.024*	0.362**	0.011	-0.259**	0.197**
	1	-0.094**	-0.980**	0.087*	-0.271**	-0.014	0.174**	-0.139**
	2	0.016	0.085	-0.110**	0.116	-0.013	0.086	0.095*
	3	-0.030	0.210	0.113*	-0.167	0.009	0.026	-0.080
	4	-0.049	-0.022	-0.058	-0.027	0.015	-0.034	-0.006
Addendum								
Sum of Q’ coefficients		0.042	0.007	.007	0.011	0.008	-0.005	0.065
Long run effect of a change in Q’		0.307	0.055	0.055	0.082	0.058	-0.041	0.481

All variables are in the following form: Y’ = ln(Y/Y*), where Y is a given variable, and Y* its estimated trend.
*Significant at the 5 percent level **Significant at the 1 percent level

Table 4: Regression Results for Components of GDP Identity, 1948:2-2001:3

		Dependent Variable						
Independent Variable	**Lag length, s**	**Employment Rate, r**	**Product-ivity, z**	**Participation rate, f**	**Average hours, h**	**Population, n**	**Output mix, m^q**	**Employment mix, m^e**
Constant		-0.000	0.000	-0.000	-0.000	-0.000	0.000	-7.67E-05
Lagged dependent variable	1	1.238**	1.170**	0.994*	0.979**	1.236**	0.795**	0.936**
	2	-0.453**	-0.076	-0.073	-0.055	-0.056	0.178*	-0.059
	3	0.107	-0.158	0.075	0.057	-0.074	-0.129	0.116
	4	0.035	0.048	0.007	-0.013	-0.113	0.054	-0.088
Output ratio, Q’	0	0.208**	0.726**	-0.011*	0.335**	0.007	-0.268**	0.159**
	1	-0.111**	-1.064**	0.082**	-0.301**	-0.003	0.210**	-0.123**
	2	-0.005	0.175	-0.116*	0.081	-0.014	0.071	0.094*
	3	-0.031	0.141	0.098	-0.075	-0.002	-0.006	-0.062
	4	-0.035	0.033	-0.045	-0.044	0.021*	-0.011	-0.020
Addendum								
Sum of Q’ coefficients		0.024	0.012	0.007	0.005	0.008	0.004	0.047
Long run effect of a change in Q’		0.219	0.114	0.071	0.046	0.080	0.004	.427

All variables are in the following form: Y’ = ln(Y/Y*), where Y is a given variable, and Y* its estimated trend.
*Significant at the 5 percent level **Significant at the 1 percent level

Table 5: Summary of Results

	Long run effect of a change in Q'	
Variable	*1948:2-1979:3*	*1948:2-2001:3*
Employment Rate	0.308	0.220
Productivity	0.055	0.114
Participation	0.055	0.071
Average Hours	0.083	0.046
Population	0.058	0.080
Output Mix	-0.041	0.004
Employment Mix	0.048	0.428

The estimated Okun coefficient for the time period ending in 1979 says that each extra percentage point increase in the ratio of output to its trend—essentially a *decrease* in the output gap—is associated with a 0.3 point decrease in the employment gap—that is to say, a 0.3 point increase in employment towards its natural rate. The estimated Okun coefficient for the time period ending in 2001 says that each extra percentage point decrease in the output gap is associated with a 0.2 point decrease in the employment gap.

The changes undergone by this variable are intuitively sensible. Recall the recovery of productivity growth during the 90's shown on Table 1, and productivity's increasing share of the total contributions to potential output growth since its drop-off in the 70's, as shown in Figure 2. Increased productivity yields higher levels of *potential* output. A higher level of potential output implies that "full" employment will generate more actual output than if the increased productivity had not occurred. From a policy standpoint, the other side of this coin is that it will require *larger* increases in output to reach the same employment targets. The results reported above are consistent with this: in 2001, the same increase in output towards its potential is associated with a lesser increase in employment towards its natural rate than would have occurred in 1979. Thus, the implication is that the recent seemingly slow recovery of employment despite GDP growth that has long since recovered does not represent a deviation from historical norms, but perhaps is an indication that that norm has changed.

Accompanying the decrease in the ratio between output and unemployment is an *in*crease over time in the ratio between output and productivity. The estimated long run effect of a change in the output gap on the deviation from trend of productivity changes from 0.055 to 0.114 between the first and second sets of regressions. For the time period ending in 1979, each extra percentage point decrease in the output gap—as the ratio of GDP to its trend rises—is associated with a 0.055 point decrease in the productivity "gap"—that is, productivity rises 0.055 point towards its trend. For the time period ending in 2001, each extra percentage point decrease in the output gap is associated with a 0.11 decrease in the productivity "gap." In Gordon's original, as reported in Table 2, the long run effect of a change in Q' on productivity was negligibly small (0.014), leading Gordon to conclude that there was "virtually no permanent productivity bonus to be enjoyed from a period of high utilization of the economy's resources" and that there is only a "transitory productivity bulge" associated with an increase in the output ratio (Gordon, 1984). Not only do this study's results for the period ending in 1979 differ from his—estimating the long run effect of a change in the output ratio on productivity to be .055—but also that long run effect appears to have increased in magnitude over time. The "negligible" long term productivity bonus found by Gordon and only ambiguously contradicted by this study's results for the same time period seems to have grown by 2001 into a modest but still significant long term effect. The implications of this are that the long term increased productivity associated with an increase in the output ratio will work via the process described above to ultimately affect the output-employment relationship.

Although the focus of this study has been the behavior of the employment and productivity variables relative to output, the estimated results for the other components of the output identity are worth mentioning. The small long run effect of a change in *Q'* on labor force participation remains relatively consistent between the first and second sets of regressions. Likewise, the long run effect on the employment mix and population variables remained consistent. The long run effect on the output mix variable went from –0.041 in 1979 to 0.041 in 2001, inexplicably switching signs, but over the span of such a small magnitude as to be of little concern. Finally, the long run effect of a change in *Q'* on average hours went from 0.08 in the first set of regressions to 0.05 in the second set, meaning that over the period ending in 2001 the same increase in the output ratio yields a smaller increase in the ratio of average hours to its trend, a finding consistent with the behavior of the employment variable: the same productivity gains that appear to have affected the output-employment ratio would have the same effect on average hours.

V. Conclusion

The starting point for this study was employment growth that seemed too slow to be consistent with the historical norms predicted by Okun's Law. Indeed, the results reported above suggest that employment does respond more slowly now than in the past to changes in output—changing from an approximate 1-to-3 relationship in the regressions for the time period ending in 1979 to an approximate 1-to-5 relationship for the time period ending in 2001. Further investigation into the stability of Okun's Law would need to be carried out, but it appears that it may not be as stable in the long run as it is thought to be. Recent job growth reports reinforce this interpretation: November 2005 marked the end of the fourth year of the worst recovery on record for all recoveries that have lasted at least this long (Bernstein, 2005). Job growth has indeed remained positive since June 2003, but in the past four years payrolls have increased only by 2.6%. Over the same number of years the recoveries which began in 1975, 1982, and 1991 saw job growth of 16.7%, 13%, and 7.6% respectively (Bernstein, 2005). These numbers are consistent with the findings of this paper. Furthermore, the finding—contradictory to Gordon's—that there is a small but permanent productivity bonus resulting from an increase in the output ratio may indicate a possible cause for the changing relationship between output and employment.

Appendix A: Estimation of Trends for Components of Output Identity, 1948-2005

Table A1: GDP Trend Estimate

Variable	Coefficient	t-stat
C	1751.587	56.792
Time	10.069	15.900**
Time-squared	0.128	47.014**

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.996
F-Statistic: 31026.39
Probability F: 0.000

Figure A1: GDP Trend Estimate

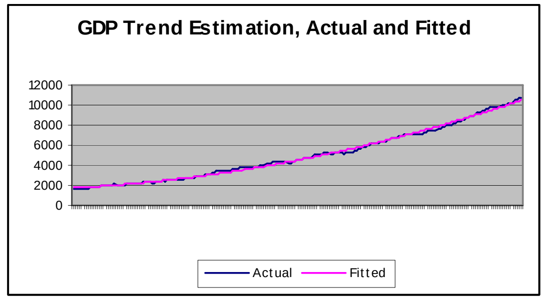


Table A2: Employment Rate Trend Estimate

Variable	Coefficient	t-stat
C	0.965	371.268
Time	-0.000	-8.303**
Time-squared	1.65E-06	7.167**

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.259
F-Statistic: 40.476
Probability F: 0.000

Figure A2: Employment Trend Estimate

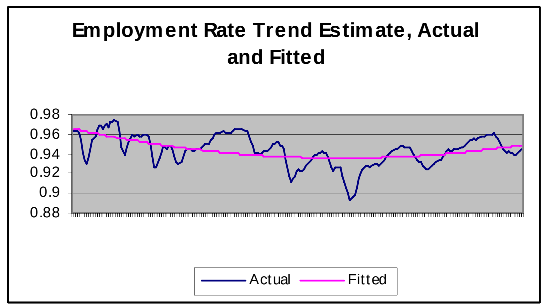


Table A3: Employment Mix Trend Estimate

Variable	Coefficient	t-stat
C	0.774	274.363
Time	0.001	24.115**
Time-squared	-2.80E-06	-11.211**

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.928
F-Statistic: 1458.505
Probability F: 0.000

Figure A3: Emp. Mix Trend Estimate

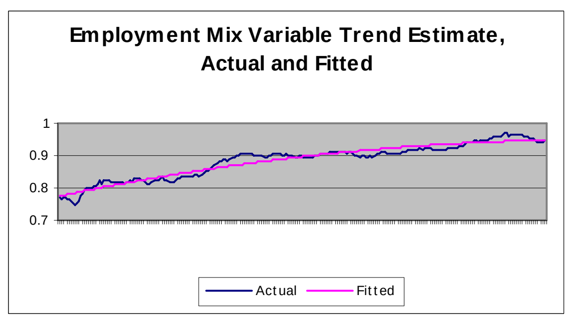
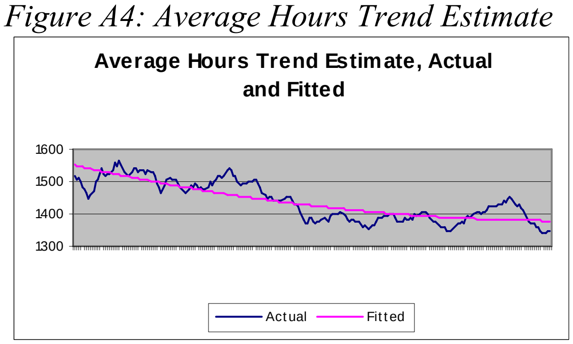


Table A4: Average Hours Trend Estimate

Variable	Coefficient	t-stat
C	1550.674	232.795
Time	-1.463	-10.703**
Time-squared	0.003	5.298**

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.697

Figure A4: Average Hours Trend Estimate



F-Statistic: 260.611
Probability F: 0.000

Table A5: Participation Trend Estimate

Variable	Coefficient	t-stat
C	58.078	313.932
Time	0.025	6.592**
Time-squared	9.56E-05	5.851**

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.914
F-Statistic: 1209.031
Probability F: 0.000

Figure A5: Participation Trend Est.

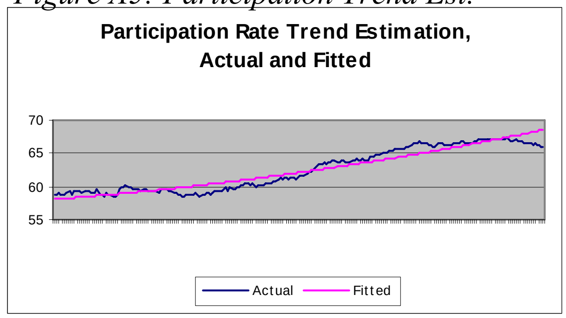


Table A6: Productivity Trend Estimate

Variable	Coefficient	t-stat
C	13.202	73.579
Time	0.090	24.545**
Time-squared	0.000	9.661**

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.987
F-Statistic: 9163.511
Probability F: 0.000

Figure A6: Productivity Trend Estimate

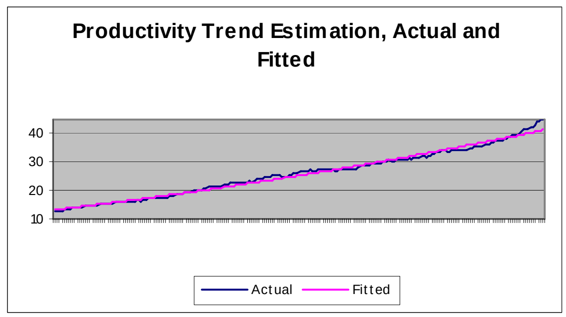


Table A7: Population Trend Estimate

Variable	Coefficient	t-stat
C	96420.83	168.906
Time	446.397	38.082**
Time-squared	0.536	10.639**

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.993
F-Statistic: 18629.42
Probability F: 0.000

Figure A7: Population Trend Estimate

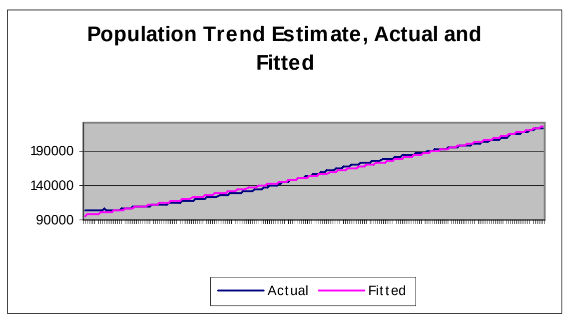
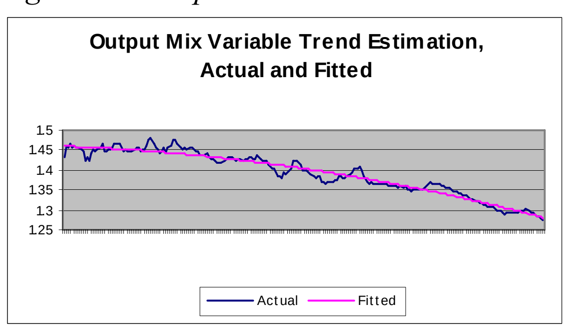


Table A8: Output Mix Trend Estimate

Variable	Coefficient	t-stat
C	1.460	580.288
Time	-0.000	-3.977**
Time-squared	-2.63E-06	-11.836**

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.945
F-Statistic: 1960.371
Probability F: 0.000

Figure A8: Output Mix Trend Estimate



Appendix B: Regression Results for 1948:2 through 2001:3

Table B1: Regression for Employment Rate, R'

Variable	Coefficient	t-stat
C	-0.000	-1.188
Q'	0.208	12.416**
Q' (-1)	-0.111	-3.940**
Q' (-2)	-0.005	-0.202
Q' (-3)	-0.031	-1.066
Q' (-4)	-0.035	-1.573
R' (-1)	1.238	17.773**
R' (-2)	-0.4531	-4.025**
R' (-3)	0.107	0.993
R' (-4)	0.035	0.597

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.975
Durbin-Watson Stat: 1.985
F-Statistic: 980.679
Probability F: 0.000

Table B2: Regression for Productivity, Z'

Variable	Coefficient	t-stat
C	0.000	1.015
Q'	0.726	17.846**
Q'(-1)	-1.064	-11.915**
Q'(-2)	0.175	1.484
Q'(-3)	0.141	1.227
Q'(-4)	0.033	0.521
Z'(-1)	1.170	16.646**
Z'(-2)	-0.076	-0.703
Z'(-3)	-0.158	-1.480
Z'(-4)	0.048	0.749

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.969
Durbin-Watson Stat: 1.965
F-Statistic: 793.170
Probability F: 0.000

Table B3: Regression for Participation Rate, F'

Variable	Coefficient	t-stat
C	-0.000	-1.243
Q'	-0.011	-0.490
Q' (-1)	0.082	2.216*
Q' (-2)	-0.116	-3.072**
Q' (-3)	0.098	2.563*
Q' (-4)	-0.045	-1.935

F' (-1)	0.994	14.614*
F'(-2)	-0.073	-0.771
F'(-3)	0.075	0.812
F'(-4)	0.007	0.109

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.954
Durbin-Watson Stat: 2.022
F-Statistic: 511.680
Probability F: 0.000

Table B4: Regression for Average Hours, H'

Variable	Coefficient	t-stat
C	-0.000	-0.611
Q'	0.335	9.421**
Q' (-1)	-0.301	-5.139**
Q' (-2)	0.081	1.313
Q' (-3)	-0.075	-1.216
Q' (-4)	-0.044	-1.043
H'(-1)	0.979	14.221**
H'(-2)	-0.055	-0.571
H'(-3)	0.057	0.600
H'(-4)	-0.013	-0.203

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.957
Durbin-Watson Stat: 2.008
F-Statistic: 556.101
Probability F: 0.000

Table B5: Regression for Population, N'

Variable	Coefficient	t-stat
C	-0.000109	-1.112926
Q'	0.007738	0.733641
Q' (-1)	-0.003675	-0.219924
Q' (-2)	-0.014429	-0.866755
Q' (-3)	-0.002406	-0.144201
Q' (-4)	0.021649	2.047606*
Ň'(-1)	1.236798	18.26584**
Ň'(-2)	-0.056311	-0.516172
Ň'(-3)	-0.074287	-0.679026
Ň'(-4)	-0.113161	-1.708413

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.994970
Durbin-Watson Stat: 2.006904
F-Statistic: 4857.864
Probability F: 0.0000

Table B6: Regression for Output Mix, M^q

Variable	Coefficient	t-stat
C	0.000	0.545
Q’	-0.268	-12.860**
Q’ (-1)	0.210	5.620**
Q’ (-2)	0.071	1.780
Q’ (-3)	-0.006	-0.154
Q’ (-4)	-0.011	-0.451
M^q (-1)	0.795	11.611**
M^q (-2)	0.178	2.055*
M^q (-3)	-0.129	-1.589
M^q (-4)	0.054	0.922

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.901
Durbin-Watson Stat: 1.898
F-Statistic: 225.940
Probability F: 0.000

Table B6: Regression for Output Mix, M^q

Variable	Coefficient	t-stat
C	-7.67E-05	-0.310
Q’	0.159	5.893**
Q’ (-1)	-0.123	-2.796**
Q’ (-2)	0.094	2.108*
Q’ (-3)	-0.062	-1.377
Q’ (-4)	-0.020	-0.684
M^e (-1)	0.936	13.741**
M^e (-2)	-0.059	-0.637
M^e (-3)	0.116	1.270
M^e (-4)	-0.088	-1.395

* Significant at the 5 percent level
**Significant at the 1 percent level
Adjusted R-Squared: 0.901
Durbin-Watson Stat: 1.898
F-Statistic: 225.940
Probability F: 0.000

VI. References

Berstein, Jared. “Jobs Picture, December 2, 2005.” Economic Policy Institute. <http://www.epi.org> Accessed: January 3, 2006.

Blanchard, Olivier and Lawrence Katz. “What we know and do not know about the natural rate of unemployment.” *NBER Working Paper Series*. National Bureau of Economic Research, November 1996.

Clark, Peter K. "Potential GNP in the United States, 1948-1980." <research.stlouisfed.org/publications/review/77/12/1277pc.pdf>

Espinosa-Vega, Marco and Steven Russell. "History and Theory of the NAIRU: A Critical Review." *Federal Reserve Bank of Atlanta Economic Review*. Vol. 1997, No. 2 (1997), 4-25.

Gordon, Robert J. "Inflation, Flexible Exchange Rates, and the Natural Rate of Unemployment." Martin Bailey, ed., *Workers, Jobs, and Inflation*. The Brookings Institution (1982), 89-158.

Gordon, Robert J. "Unemployment and Potential Output in the 1980s." *Brookings Papers on Economic Activity*, Vol. 1984, No. 2 (1984), 537-568.

Okun, Arthur. "Potential GNP: Its Measurement and Significance." *The Political Economy of Prosperity*, Washington, D.C.: The Brookings Institution (1970), 132-145.

U.S. Department of Commerce (2004). *Bureau of Economic Analysis Data*. www.bea.doc.gov.

U.S. Department of Labor (2004). *Bureau of Labor Statistics*. http://data.bls.gov.

VII. Endnotes

[1] Gordon's benchmark quarters were as follows: 1948:4, 1953:4, 1957:3, 1960:1, 1970:3, 1974:2, 1979:3 (Gordon, 1984)